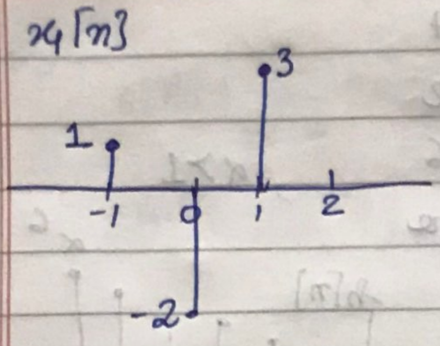
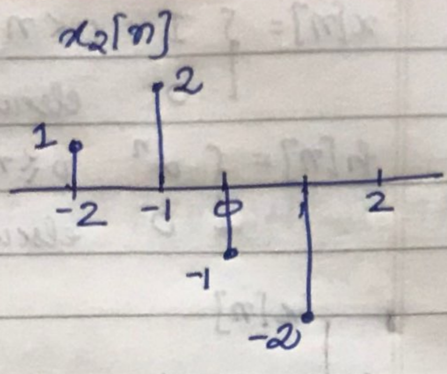
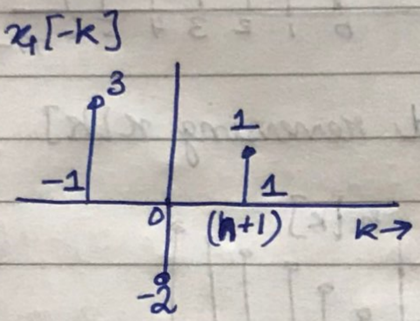
Tutorial 3 Solutions

Q1.







$0 \qquad\qquad n+1 < -2$

$1 = 1\cdot 1 \qquad\qquad n+1 = -2$

$0 = -2+2 = -2\times 1 + 1\times 2 \qquad\qquad n+1 = -1$

$-2 = 3-4-1 = 1\times 3 + -2\times 2 + 1\times -1 \qquad\qquad n+1 = 0$

$6 = 6+2-2 = 2\times 3 + -1\times -2 + 1\times -2 \qquad\qquad n+1 = 1$

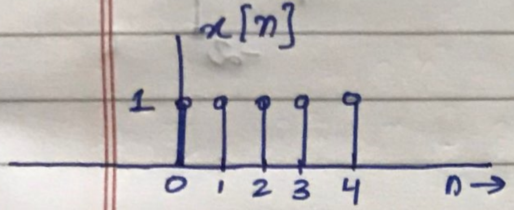
$1 = -3+4 = 3\times -1 + -2\times -2 \qquad\qquad n+1 = 2$

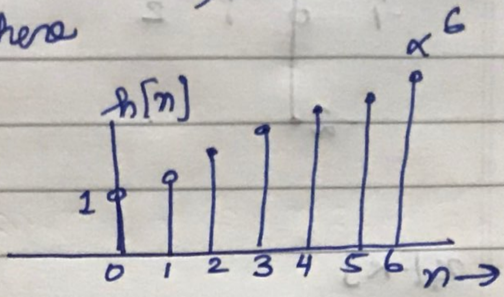
$-6 = -2\times 3 \qquad\qquad n+1 = 3$

$$\therefore y[n] = \begin{cases} 0 & n < -3 \\ 1 & n = -3 \\ 0 & n = -2 \\ -2 & n = -1 \\ 6 & n = 0 \\ 1 & n = 1 \\ -6 & n = 2 \\ 0 & n > 2 \end{cases}$$

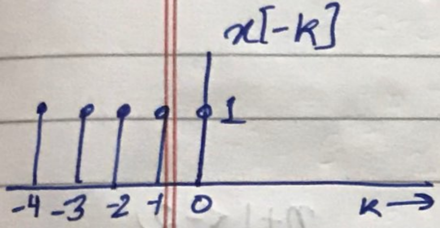
Q2. $x[n] = \begin{cases} 1 & 0 \leq n \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

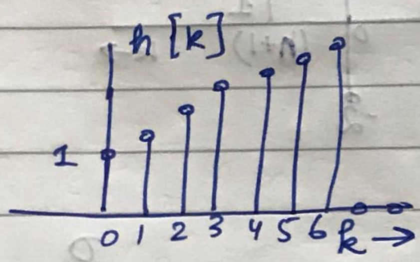
$h[n] = \begin{cases} \alpha^n & 0 \leq n \leq 6 \\ 0 & \text{elsewhere} \end{cases}$, $\alpha > 1$





Plotting on k axis and reversing x[k].





$$y[n] = \begin{cases} 0 & n < 0 \\ \sum_{k=0}^{n} \alpha^k = \dfrac{\alpha^{n+1}-1}{\alpha-1} & 0 \leq n \leq 4 \\ \sum_{k=n-4}^{n} \alpha^k = \dfrac{\alpha^{n-4}(1-\alpha^5)}{(1-\alpha)} & 4 < n < 6 \\ \sum_{k=n-4}^{6} \alpha^k & 6 < n < 10 \end{cases}$$

Let $n-4 = m+k$, $m = (n-4)-k$

if $k = n-4$ $m = 0$, if $k = 6$ $m = n-10$

$$= \sum_{m=0}^{n-10} \alpha^{n-4-m} = \sum_{m=0}^{n-10} \alpha^{n-4}\alpha^{-m}$$

$$= \sum_{n=0}^{10-n} \alpha^m \alpha^{n-4} = \alpha^{n-4}\left[\frac{1-\alpha^{11-n}}{1-\alpha}\right]$$

Q3. $h[n] = \begin{cases} -2\sqrt{2} & n = -1, 1 \\ 4\sqrt{2} & n = -2, 2 \end{cases}$

$x[n] = e^{\frac{j\pi n}{4}}$

$y[n] = ?$

We can write $h[n]$ as follows :

$h[n] = -2\sqrt{2}\,\delta[n+1] - 2\sqrt{2}\,\delta[n-1] + 4\sqrt{2}\,\delta[n+2] + 4\sqrt{2}\,\delta[n-2]$

$$y[n] = x[n] * h[n]$$
$$= x[n] * \left[-2\sqrt{2}\,\delta[n+1] - 2\sqrt{2}\,\delta[n-1] + 4\sqrt{2}\,\delta[n+2] + 4\sqrt{2}\,\delta[n-2]\right]$$
$$= -2\sqrt{2}\,x[n+1] - 2\sqrt{2}\,x[n-1] + 4\sqrt{2}\,x[n+2] + 4\sqrt{2}\,x[n-2]$$

$x[n] = e^{\frac{j\pi n}{4}}$, $x[n+1] = e^{\frac{j\pi(n+1)}{4}}$, $x[n-1] = e^{\frac{j\pi(n-1)}{4}}$,

$x[n+2] = e^{\frac{j\pi(n+2)}{4}}$, $x[n-2] = e^{\frac{j\pi(n-2)}{4}}$

$$\therefore\ y[n] = -2\sqrt{2}\,e^{\frac{j\pi(n+1)}{4}} - 2\sqrt{2}\,e^{\frac{j\pi(n-1)}{4}} + 4\sqrt{2}\,e^{\frac{j\pi(n+2)}{4}} + 4\sqrt{2}\,e^{\frac{j\pi(n-2)}{4}}$$

$$= 2\sqrt{2}\,e^{\frac{j\pi n}{4}}\left[e^{\frac{j\pi}{4}} + e^{\frac{-j\pi}{4}}\right] + 4\sqrt{2}\left[e^{\frac{j\pi}{2}} + e^{\frac{-j\pi}{2}}\right] e^{\frac{j\pi n}{4}}$$

$$= -2\sqrt{2}\,e^{\frac{j\pi n}{4}}\left[\frac{2}{\sqrt{2}}\right] + 4\sqrt{2}\,[0]$$

$$= -4e^{j\pi n/4}$$

Q4. $y[n]+2y[n-1]=x[n]-2x[n-2]$

$$\therefore y[n]=x[n]-2x[n-2]-2y[n-1]$$

Given initially at rest $\therefore$ before input, the output is 0

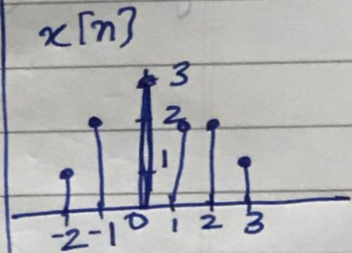
$\therefore y[n]=0$ for $n<-2$

Now finding $y[n]$ for different values of n, $(n>-2)$

$n=-2$ $\quad y[-2]=x[-2]-2x[-4]-2y[-3]$

$y[-2]=1-0-0=1$

$\therefore y[-2]=1$



$n=-1$ $\quad y[-1]=x[-1]-2x[-3]-2y[-2]$

$=2-0-2\times1$

$=2-2=0$

$\therefore y[-1]=0$

$n=0$ $\quad y[0]=x[0]-2x[-2]-2y[-1]$

$=3-2\times1-2\times0=3-2=1$

$\therefore y[0]=1$

$n=1$ $\quad y[1]=x[1]-2x[-1]-2y[0]$

$=2-2\times2-2\times1=2-4-2=-4$

$n=2$ $\quad y[2]=x[2]-2x[0]-2y[1]$

$=2-2\times3-2\times(-4)$

$=2-6+8=-4+8=4$

$y[2]=4$

$n=3$ $\quad y[3]=x[3]-2x[1]-2y[2]=1-2\times2-2\times4=-13$

$n=4$ $\quad y[4]=x[4]-2x[2]-2y[3]=0-2\times2-2(-13)$

$=-4+26=22$

$n=5$ $\quad y[5] = x[5] - 2x[3] - 2y[4]$

$= 0 - 2\times1 - 2(22)$

$= -2 - 44 = -46$

$n=6$ $\quad y[6] = x[6] - 2x[4] - 2y[5]$

$= 0 - 0 - 2(-46)$

$= (-2)(-46)$

$n=7$ $\quad y[7] = (-2)^2(-46)$

$\vdots$

$y[n] = (-46)(-2)^{n-5}$ for $n \geq 7$

Q5. (i)

$$s[n] = \sum_{k=-\infty}^{\infty} a_k \, u[n-k]$$

$$s[n] = u[n-4] - 4u[n-2] + 4u[n+1] - u[n+3]$$

(ii) $$s[n] = \sum_{k=-\infty}^{\infty} a_k \, \delta[n-k]$$

$$= -\delta[n+3] - \delta[n+2] + 3\delta[n+1] + 3\delta[n] + 3\delta[n-1] - \delta[n-2] - \delta[n-3]$$