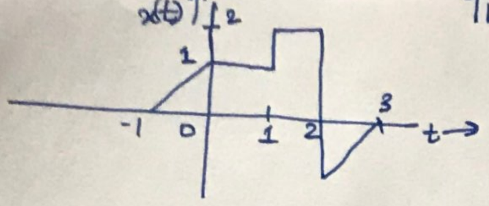
Tutorial Sheet 1 · Solutions

Q1.



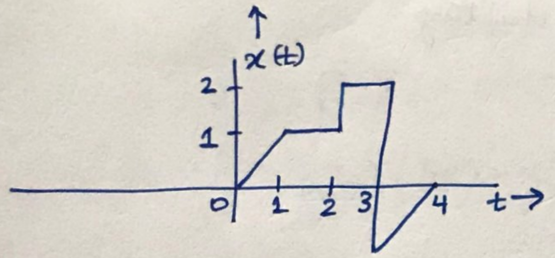
a) $x(t-1)$

time shift

$x(t)$

$t \to t-1$

$x(t-1)$



b) $x(4-t/2) = x(-t/2+4)$

$x(t)$

$t \to t+4$ time shift

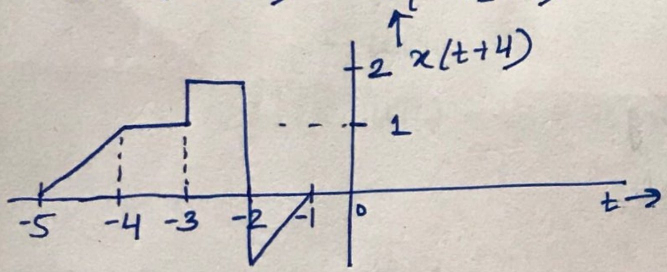
$x(t+4)$

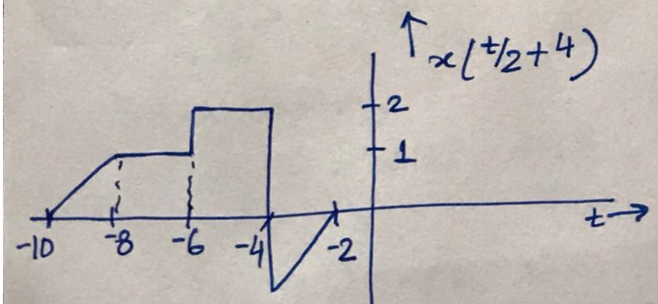
$t \to t/2$ scale (expansion)

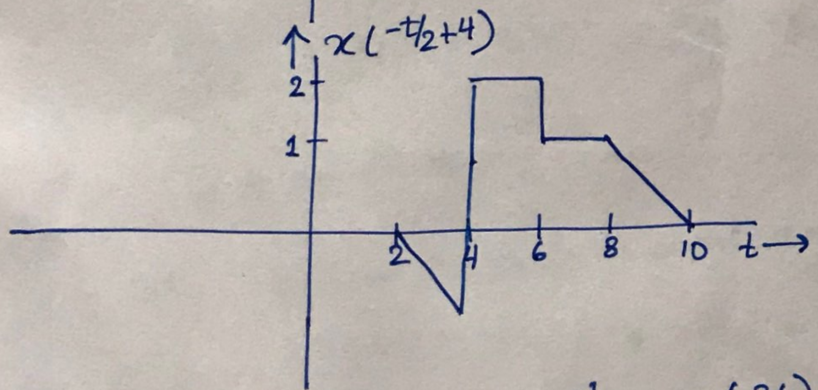
$x(t/2+4)$

$t \to -t$

$x(-t/2+4)$ reversal.

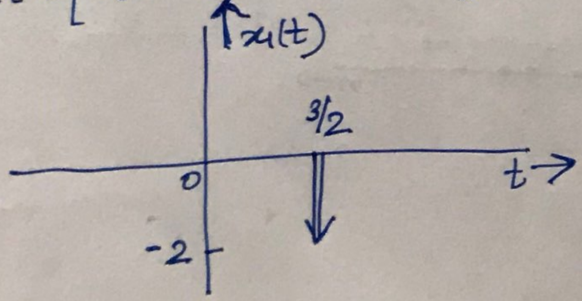




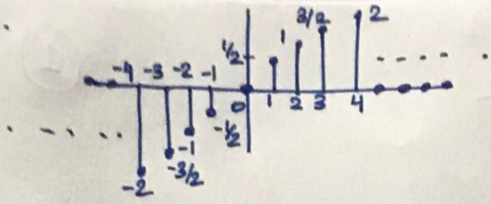


c) $x(t)\left[\delta(t+3/2) - \delta(t-3/2)\right] = x(-3/2)\delta(t+3/2) - x(3/2)\delta(t-3/2)$

$= x_1(t)$



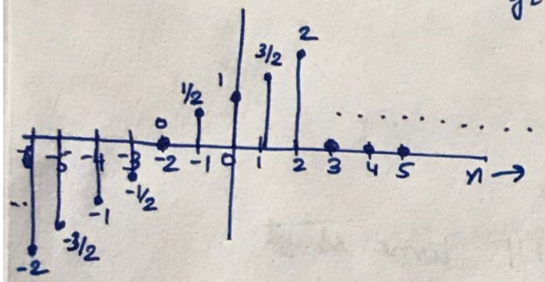
Q2.



a) $y[n+2] + y[-n-1]$

$\underline{y[n+2]}$

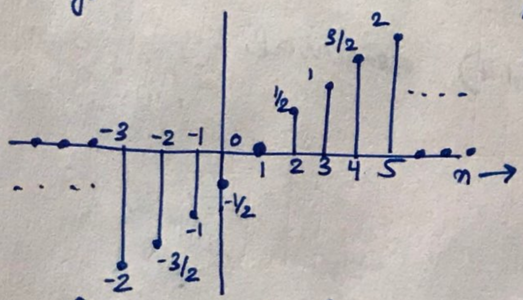
$y[n]$
$n \rightarrow n+2$ ~~[illegible]~~ shifting
$y[n+2]$

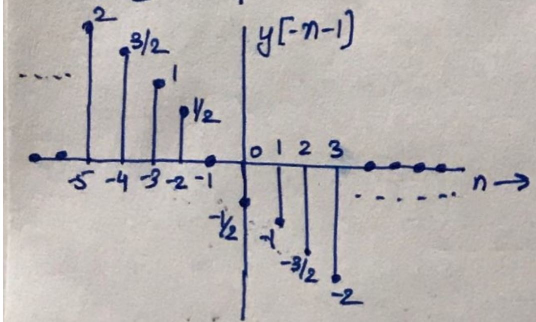


$\underline{y[-n-1]}$

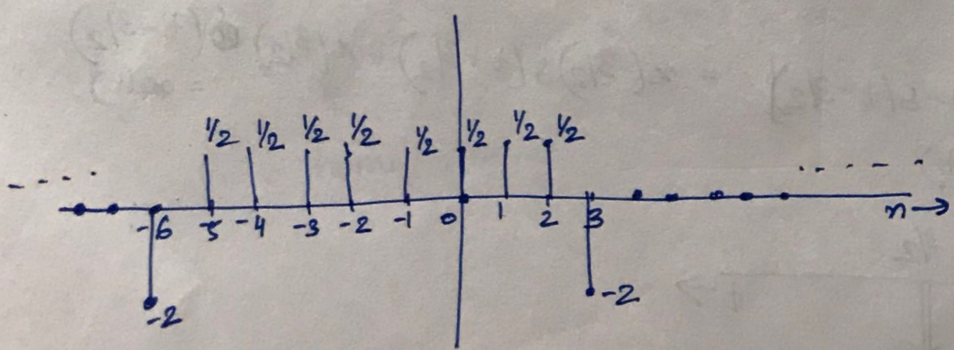
$y[n]$
$n \rightarrow n-1$ shifting
$y[n-1]$
$n \rightarrow -n$ reversal
$y[-n-1]$

$y[n-1]$



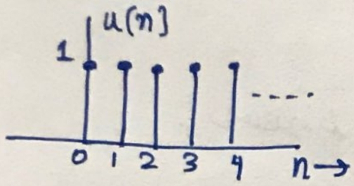


$\underline{y[n+2] + y[-n-1]}$

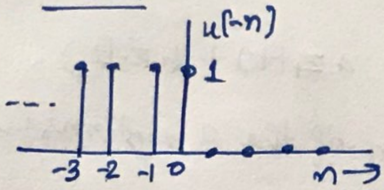


b) $y[n+1]\,(u[n+3]-u[-n])$

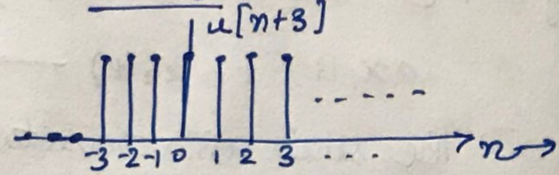
$u[n]$



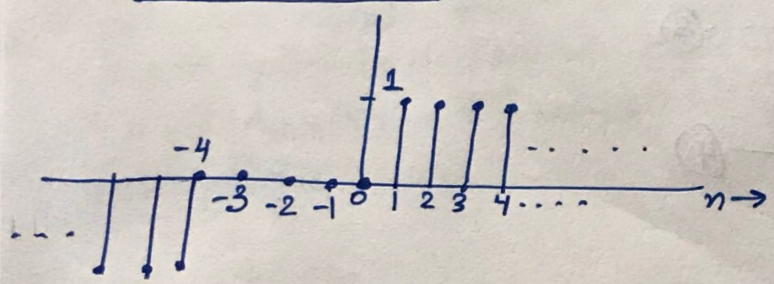
$u[-n]$



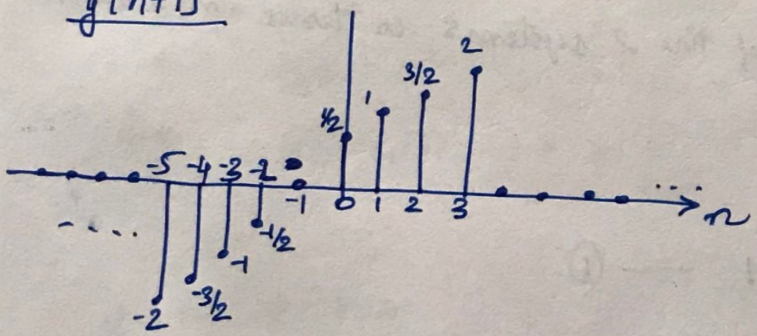
$u[n+3]$



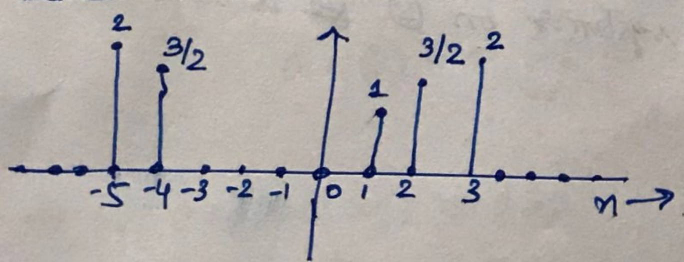
$u[n+3]-u[-n]$



$y[n+1]$

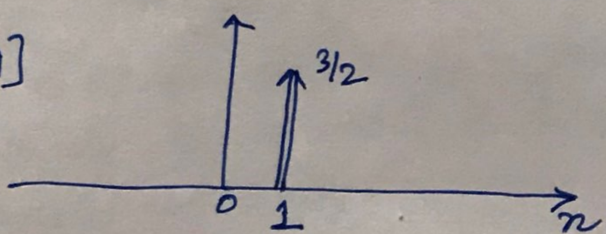


$y[n+1]\,[u[n+3]-u[-n]]$

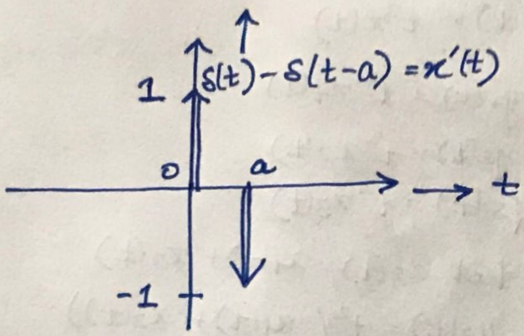


c) $y[3n]\,\delta[n-1]$

$y[3\cdot 1]\delta[n-1] = y[3]\,\delta[n-1]$



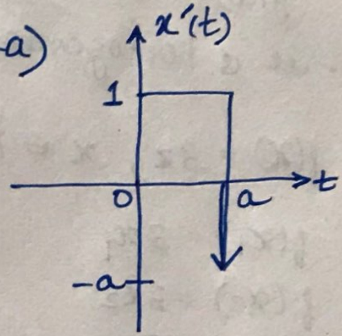
Q3. a) $x(t) = u(t) - u(t-a)$, $a>0$



$$x'(t) = u'(t) - u'(t-a)$$
$$= \delta(t) - \delta(t-a)$$

b) $x(t) = t[u(t) - u(t-a)]$, $a>0$

$$x'(t) = \left(\frac{dt}{dt}\right)(u(t) - u(t-a)) + t\frac{d}{dt}[u(t) - u(t-a)]$$
$$= u(t) - u(t-a) + t\,\delta(t) - t\,\delta(t-a)$$
$$= u(t) - u(t-a) + 0\cdot\delta(t) - a\cdot\delta(t-a)$$
$$\therefore x'(t) = u(t) - u(t-a) - a\,\delta(t-a)$$



Q4. a) $\frac{dy(t)}{dt} = x(t)$

Let us consider,

$$\frac{dy_1(t)}{dt} = x_1(t), \quad \frac{dy_2(t)}{dt} = x_2(t), \quad \frac{dy_3(t)}{dt} = x_3(t)$$

Let $x_3(t) = x_1(t) + x_2(t)$

$$\frac{dy_3(t)}{dt} = x_1(t) + x_2(t) = \frac{dy_1(t)}{dt} + \frac{dy_2(t)}{dt} = \frac{d(y_1(t)+y_2(t))}{dt}$$

$\therefore$ it is additive.

Let $y_1(t)$ corresponds to input $a\,x(t) = x_1(t)$

$$\frac{dy_1(t)}{dt} = x_1(t) = a\,x(t) = a\frac{dy(t)}{dt} = \frac{d(a\,y(t))}{dt}$$

$\therefore$ it is homogeneous.

b) $y(t) = t^2 x(t)$

$y_1(t) = t^2 x_1(t)$

$y_2(t) = t^2 x_2(t)$

$y_3(t) = t^2 x_3(t)$

Let $x_3(t) = x_1(t) + x_2(t)$

$y_3(t) = t^2 (x_1(t) + x_2(t))$

$= t^2 x_1(t) + t^2 x_2(t)$

$= y_1(t) + y_2(t)$

$\therefore$ it is additive

Let $y_1(t)$ ~~[illegible]~~ correspond to input $x_1(t) = a\, x(t)$

$y_1(t) = t^2 x_1(t) = t^2 a\, x(t) = a\, t^2 x(t) = a\, y(t)$

$\therefore$ it is homogeneous.

c) $f(x) = 3x, \quad x \in \{0, 2, 4, 6, \dots\}$

$f(x_1) = 3x_1$

$f(x_2) = 3x_2$

$f(x_3) = 3x_3$

let $x_3 = x_1 + x_2$

$f(x_3) = 3(x_1 + x_2) = f(x_1) + f(x_2)$

$\therefore f(x_1 + x_2) = f(x_1) + f(x_2) \quad \therefore$ additive

$f(\underbrace{\alpha x}_{x_1})$ exists only if $x_1 = \alpha x \in \{0, 2, 4, 6, \dots\}$

The function αx is ~~undef~~ not defined for all values of αx.

eg. if $\alpha = 1.3$, then $f(\alpha x)$ is undefined.

$\therefore$ not homogeneous.

Q5. a) $y(t) = 3x^2(t)\, u(t)$

for input $x(t - t_0)$ output $y_1(t) = 3x^2(t - t_0)\, u(t)$

for $y(t - t_0) = 3x^2(t - t_0)\, u(t - t_0)$

as $y_1(t) \neq y(t - t_0) \quad \therefore$ Not time invariant.

b) $y(t) = x^2(t) + x(t)$

For input $x(t-t_0)$, $y_1(t_1) = x^2(t-t_0) + x(t-t_0)$

For $y(t-t_0) = x^2(t-t_0) + x(t-t_0)$ as $y_1(t_1) = y(t-t_0)$

$\therefore$ Time invariant.

c) $y[n] = n\,x[n]$

For input $x[n-n_0]$, $y_1[n_1] = n\,x[n-n_0]$

For $y[n-n_0] = [n-n_0]\,x[n-n_0]$

as $y_1[n_1] \neq y[n-n_0]$ $\therefore$ not time invariant.

Q6. $y[n] = x[n]\{g[n] + g[n-1]\}$ $\qquad x[n] \rightarrow \boxed{S} \rightarrow y[n]$

a) if $g[n] = 1 \ \forall n$

then $y[n] = x[n]\{1+1\} = 2\,x[n]$

For input $x[n-n_0]$, $y_1[n_1] = 2x[n-n_0]$

For $y[n-n_0] = 2x[n-n_0]$

as $y_1[n_1] = y[n-n_0]$ $\therefore$ time invariant.

b) $g[n] = n$

$\therefore g[n-1] = n-1$

$\therefore y[n] = x[n](n+n-1) = (2n-1)\,x[n]$

For input $x[n-n_0]$, $y_1[n_1] = (2n-1)\,x[n-n_0]$

For $y_0[n-n_0] = (2(n-n_0)-1)\,x[n-n_0]$

as $y_1[n_1] \neq y[n-n_0]$ $\therefore$ not time invariant.

c) $g[n] = 1 + (-1)^n$

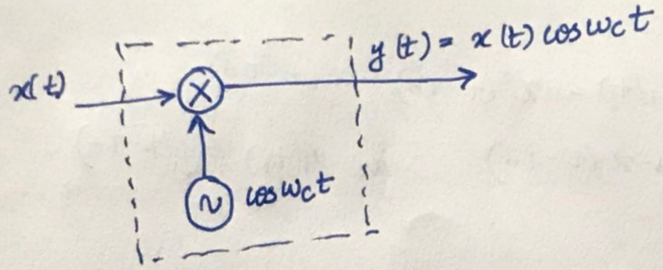
$\therefore g[n-1] = 1 + (-1)^{n-1}$

$\therefore y[n] = x[n]\left(1 + (-1)^n + 1 + (-1)^{n-1}\right) = x(n)\,[2 + \underbrace{(-1)^n + (-1)^{n-1}}_{=0}]$

$= 2x[n]$

as per part (a), it is time invariant.

Q7. a)



(i) $y_1(t) = x_1(t)\cos\omega_c t$

$y_2(t) = x_2(t)\cos\omega_c t$

$y_3(t) = x_3(t)\cos\omega_c t$

Let $x_3(t) = x_1(t) + x_2(t)$

$y_3(t) = (x_1(t) + x_2(t))\cos\omega_c t$

$= x_1(t)\cos\omega_c t + x_2(t)\cos\omega_c t$

$= y_1(t) + y_2(t)$

$\therefore$ ~~to~~ additive

(ii) $y_1(t)$ ~~be~~ correspond to $a\,x(t) = x_1(t)$

$y_1(t) = a\,x(t)\cos\omega_c t$

$y_1(t) = a\,y(t)$

$\therefore$ homogeneous.

(iii) For input $x(t-t_0)$, $y_1(t_1) = x(t-t_0)\cos(\omega_c t)$

For $y(t-t_0) = x(t-t_0)\cos(\omega_c(t-t_0))$

as $y_1(t_1) \neq y(t-t_0)$ $\therefore$ time varying.

b) $y(t) = x(\sin(t))$

(i) $y_1(t) = x_1(\sin(t))$

$y_2(t) = x_2(\sin(t))$

$y_3(t) = x_3(\sin(t))$

Let $x_3(\sin(t)) = x_1(\sin(t)) + x_2(\sin(t))$

$\therefore y_3(t) = x_1(\sin t) + x_2(\sin(t))$

$= y_1(t) + y_2(t)$

$\therefore$ additive

(ii) Let $y_1(t)$ correspond to $a\,x(\sin(t)) = x_1(\sin(t))$

$\therefore y_1(t) = a\,x(\sin(t)) = a\,y(t)$

$\therefore$ homogenous

$y(t) = x(\sin(t))$ 8

for input $x(\sin(t)-(t_0))$, $y_1(t_1) = x(\sin(t)-(t_0)) = x((\sin t) - t_0)$

For $y(t-t_0) = x(\sin(t-t_0))$

as $y_1(t_1) \neq y(t-t_0)$ $\therefore$ Not time invariant

(c) $y(t) = e^{x(t)}$

(i) $y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y_3(t) = e^{x_3(t)}$

Let $x_3(t) = x_1(t) + x_2(t)$

$y_3(t) = e^{x_1(t)+x_2(t)} = e^{x_1(t)} e^{x_2(t)} = y_1(t)\, y_2(t)$

$\therefore$ not additive

(ii) Let $y_1(t)$ correspond to $x_1(t) = a x(t)$

$y_1(t) = e^{x_1(t)} = e^{a x_1(t)} \neq a y(t)$

$\therefore$ not homogeneous.

(iii) $y(t) = e^{x(t)}$

For $x(t-t_0)$, $y_1(t_1) = e^{x(t-t_0)}$

For $y(t-t_0) = e^{x(t-t_0)}$

as $y_1(t_1) = y(t-t_0) \Rightarrow$ time invariant.

Q8. a) Let system be S such that,

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

since S is time invariant,

$$x(t-t_0) \longrightarrow \boxed{S} \longrightarrow y(t-t_0)$$

if periodic then, $x(t) = x(t-T)$ where T is time period.

by time invariance property ~~let~~ let $t_0 = T$

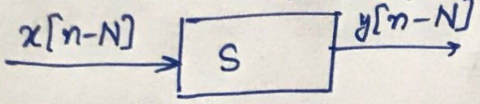
then $$x(t-T) \longrightarrow \boxed{S} \longrightarrow y(t-T)$$

$\therefore$ if $x(t) = x(t-T) \Rightarrow y(t) = y(t-T)$ Thus $y(t)$ is also periodic with period T. Hence proved.

Similar to (a)

For periodicity $x[n] = x[n-N]$ N: time period.

by time invariance property,



$\therefore$ if $x[n] = x[n-N]$

then $y[n] = y[n-N]$ with period N.

b) Let input signal $x(t) = \sin(\pi t) + t$ (non-periodic)

Let the system be defined as $y(t) = x(t) - x(t-1)$ (time invariant).

then $y(t) = \sin(\pi t) + t - \sin(\pi(t-1)) - (t-1) = 2\sin(\pi t) + 1$ which is periodic with period 2.

Q9. if additive,

$$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$$

Let $x_2(t) = -x_1(t)$

then $\underset{=0}{x_1(t) - x_1(t)} \longrightarrow \underset{=0}{y_1(t) - y_1(t)}$

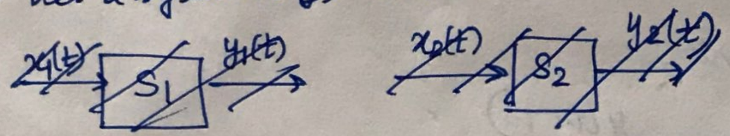
if homogenous

$$a x_1(t) \longrightarrow a y_1(t)$$

if $a = 0$

$$\underset{=0}{0 \cdot x_1(t)} \longrightarrow \underset{=0}{0 \cdot y_1(t)}$$

Q10. a) Let 2 systems be, S_1 & S_2



if $x_1(t)$ & $x_2(t)$ are inputs to S_1 then outputs are $y_1(t)$ & $y_2(t)$ respectively. Also if inputs to S_2 are $y_1(t)$ & $y_2(t)$, outputs are $z_1(t)$ & $z_2(t)$ respectively.

as the two systems are linear & time invariant,

By linearity property for S_1,

$$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t) \quad \text{—} ①$$

By linearity property for S_2,

$$a y_1(t) + b y_2(t) \longrightarrow a z_1(t) + b z_2(t) \quad \text{— ②}$$

by ① & ② we can see that,

$$a x_1(t) + b x_2(t) \longrightarrow a z_1(t) + b z_2(t)$$

∴ the series interconnection of the 2 systems is linear.

by time-invariant property for S_1,

$$x_1(t-t_0) \longrightarrow y_1(t-t_0) \quad \text{— ③}$$

by time invariant property for S_2,

$$y_1(t-t_0) \longrightarrow z_1(t-t_0) \quad \text{— ④}$$

From ③ & ④ we get,

$$x_1(t-t_0) \longrightarrow z_1(t-t_0)$$

∴ the series interconnection of the 2 systems is time-invariant.

∴ TRUE.

b) let $y(t) = x(t) + 1$ & $z(t) = y(t) - 1$ — ①

let $z(t) = x(t) + 1 - 1 = x(t)$ — ②

the 2 systems described in ① are non-linear while the series interconnection of the two systems in ② is linear.

∴ FALSE