PRP Assignment 5

1. Show that if

$$\mathrm{x}_i \geq 0, \quad E\{\mathrm{x}_i^2\} = M \quad \text{and} \quad \mathrm{s} = \sum_{i=1}^{n} \mathrm{x}_i \quad \text{then}$$
$$E\{\mathrm{s}^2\} \leq M\, E\{\mathrm{n}^2\}.$$

2. Show that if the random variables x_i are i.i.d. and normal, then their sample mean $\bar{\mathrm{x}}$ and sample variances s^2 are two independent random variables.

3. We place at random n points in the interval $(0, 1)$ and we denote by x and y the distance from the origin to the first and the last point respectively. Find $F(x)$, $F(y)$, and $F(x, y)$.

4. Let S_n be the number of successes in n Bernoulli trials (independent), where the probability of success in one trial $p = 0.4$. When $n \to \infty$, provide a value for the probability that S_n is between $\frac{n}{2} - 10$ to $\frac{n}{2} + 10$.

5. Let X_1, X_2,... be i.i.d. positive random variables with mean 5. Let Y_1, Y_2,...be i.i.d. positive random variables with mean 7. Show that

$$\frac{X_1 + X_2 + ...X_n}{Y_1 + Y_2 + ...Y_n} \to \frac{5}{7}$$

with probability 1. Does it matter whether the X_i are independent of the Y_j?

6. For i.i.d. random variables X_1, X_2,..., X_n with mean μ and variance 2, give a value of n (as a specific number) that will ensure that there is at least 98% chance that the sample mean will be within 2 standard deviations of the true mean μ.

7. Let $Y = e^X$ with X $\sim$ Exp(3). For i.i.d random variables $Y_1 + Y_2 + ...Y_n$, find the distribution of sample mean ($\overline{\mathrm{Y}}_n = \frac{1}{n}\sum_{j=1}^{n} Y_j$) when n is large.