Engineering Optimization: Lecture 1

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Course Organization

- 13 Weeks, Tue and Fri: 9:30 to 11:00 A.M
- Course Website: Coming Soon !
- Office hours: Wed: 12:00 to 1:00 p.m
- References :
  - **Convex Optimization** by Boyd and Vandenberghe http://www.stanford.edu/ boyd/cvxbook/
  - **Introduction to Convex Programming** by Y. Nesterov http://www.core.ucl.ac.be/ nesterov/Courses/INMA2460/Intro-nl.pdf
  - **Non-linear Programming** by Dimitri Bertsekas

Course Organization..

- 4 assignments/quizzes (including one possible programming assignment) – 15%
- One minor exam (open notes, your notes, no photocopies) – 25%
- 1 major exam (open notes, your notes, no photocopies) – 40%
- One course project – 20% (Only for those who satisfy Minimum requirements)
- Optimization of Grades is optimization of effort.
- Any other suggestion on evaluation?

Course Topics

- Mathematical review: Linear Algebra, Vector Calculus.
- How to frame Optimization problems?
- Convex sets and functions
- Introductory non-linear programming: Gradient descent algorithms etc.
- Convex Optimization problems: LP, QP etc.
- Duality and Lagrangian Relaxation techniques
- Algorithms for Optimizations
- Applications to Machine learning, Communications, Signal Processing etc.

What is Optimization

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to} \quad & x \in \chi \end{aligned} \tag{1}$$

where

- x is a decision variable
- f(x) is the objective function (convex, linear, non-linear..)
- χ is the feasible region (convex, non-empty)

Key questions:

- Is the problem feasible?
- How to determine if a candidate x is an optimal solution.
- How to find the optimal solution.

Applications

- Signal processing and (wireless and wireline) Communications
- VLSI
- Machine Learning and Vision
- Robotics
- Finance and Analytics
- Computational Biology
- Operations Research

Who should not take this course

- If you have a dislike for Maths – Linear Algebra, Calculus
- If you plan to attend less than 80% classes.
- If any one of the above is true.

How do I use optimization?

Questions:

- What are the possible methods to find a good solution?
- Is the problem convex?
- Does my algorithm converge to an optimal solution?
- How do I initialize my algorithm?
- What is the run-time complexity of the algorithm?

Answer: This course!

Review of Basics: Vector Calculus

- Limit
- Continuity
- Gradient

$$\nabla(f(x)) = \left[\frac{\partial f(x)}{\partial x_1} \; \frac{\partial f(x)}{\partial x_2} \dots \frac{\partial f(x)}{\partial x_N}\right]^T$$

- Taylor Series Expansion of functions
  - Expansion used to approximate complicated functions.

Norms

$f : \mathbf{R}^n \rightarrow \mathbf{R}$ is a norm if for all $x, y \in \mathbf{R}^n$, $t \in \mathbf{R}$ s

1. $f(x) \geq 0$
2. $f(tx) = |t| f(x)$
3. $f(x + y) \leq f(x) + f(y)$

$f(x)$ usually denoted as $|| \, ||_{mark}$

Example: $||l||_2$, called l_2 norm or the euclidean norm.

In general l_p, norms are $||l||_p, \;\; p \geq 1$, where,

$$||x||_p = \left(\sum_i |x_i|^p \right)^{\frac{1}{p}}$$