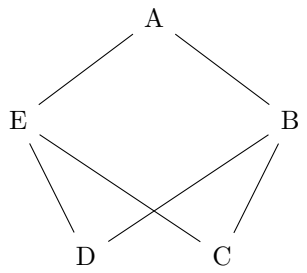
Worksheet 4
IQC 2025

February 20, 2025

1. Prove that for every Hermitian matrix A, the matrix U $= e^{-iHt}$, for some parameter t, is a unitary operator.
2. Show that the eigenstates of $U = e^{-iHt}$ for some Hermitian matrix A are the same as that of A.
3. Let E_0 denote the smallest eigenvalue of a Hermitian operator H, show that for any state $|\psi\rangle$ which is not the ground state, the expectation $\langle\psi| H |\psi\rangle \geq E_0$.
4. Write out the matrix form of the operator QFT_{2^2}.
5. Show that QFT is a unitary operator and write its inverse operator.
6. Check if $\mathrm{QFT}_{2^n} |0\rangle^{\otimes n} = (\mathrm{QFT}_2|0\rangle)^{\otimes n}$.
7. Write the following QUBO problems in their matrix forms. Also, construct the corresponding Ising Hamiltonian.
   (a) $x_1 + x_1x_2 - 3x_3 + 5$
   (b) $x_1x_2 - x_2x_3 + x_3x_4 - 5x_2 + 16x_3$
8. For two binary variables x_1 and x_2 we can model $x_1 = x_2$ as the following QUBO: $x_1 + x_2 - 2x_1x_2$. Verify the above formulation and write the corresponding Ising Hamiltonian. Also, formulate the QUBO for $x_1 \geq x_2$.
9. For the given array $A = [1, 3, 4]$. Formulate, the partition problem for the array as a QUBO and write the corresponding Ising Hamiltonian.
10. Consider the following graph G. Construct a Hamiltonian to find the Maximum cut of this graph.



11. Perform addition of the integers 5 and 2 using Draper QFT Adder circuit.
12. Verify that in the Draper QFT Adder on n qubit registers containing $|a\rangle, |b\rangle$ we obtain $(a + b) \mod 2^n$.