Worksheet 2
IQC 2025

16 January 2025

Let

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Does X map a qubit to a qubit i.e. $X|\psi\rangle$ for any arbitrary qubit $|\psi\rangle$ is also a qubit?

1. Write out the following tensor products as vectors and matrices.
   (a) $|0\rangle \otimes |1\rangle$
   (b) $(\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha^*|1\rangle + \beta^*|0\rangle)$, here α^* denotes the conjugate of α.
   (c) $H \otimes H$
   (d) $X \otimes H$
2. For the following operators X and Y, verify that $(X \otimes Y)(|a\rangle|b\rangle) = X|a\rangle \otimes Y|b\rangle$.
   (a) $H \otimes H|0\rangle|1\rangle$
   (b) $X \otimes X|1\rangle|1\rangle$
   (c) $X \otimes I \otimes H|0\rangle|1\rangle\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)$
3. Given two normalised states $|\psi\rangle$, $|\phi\rangle$, show that $|\psi\rangle \otimes |\phi\rangle$ is also normalised.
4. Prove that
$$H^{\otimes n}|\mathbf{x}\rangle = \sum_{y \in \{0,1\}^n} (-1)^{\mathbf{x} \cdot y}|y\rangle$$
5. Write the measurement outcomes when the following two qubit states are measured in the standard as well as the Hadamard basis?
   (a) $\frac{1}{\sqrt{3}}(|00\rangle + |10\rangle + |11\rangle)$
   (b) $\frac{1}{\sqrt{2}}(|{+}{+}\rangle + |{-}{-}\rangle)$
6. Show that Hermitian and Unitary operators are normal operators.
7. Show that for Unitaries U_1, U_2 their product $U_1 U_2$ is also a unitary.
8. Show that the tensor product $U_1 \otimes U_2$ is unitary if U_1, U_2 are unitary.
9. Show that the state $|\beta_{00}\rangle$ is entangled.
10. Analyse the outcomes of the measurement of the first qubit in the Bell states in Hadamard as well as the standard basis.