Worksheet 1
IQC 2025

9 January 2025

1. Convert the given invalid quantum states into valid ones.
   (a) $(3+5i)\,|0\rangle + (1-7i)\,|1\rangle$
   (b) In vector notation, $x_1 = [-1, 1]^T$ and $x_2 = [1, -1]^T$.
   (c) $|+\rangle + |1\rangle - 3/5\,|-\rangle$
2. Find the inner product of the following states:
   (a) $|u\rangle = \sqrt{\frac{7}{11}}\,|0\rangle + \frac{2}{\sqrt{11}}\,|1\rangle$ and $|v\rangle = |+\rangle$
   (b) $|u\rangle = |+\rangle$ and $|v\rangle = \frac{8}{9}\,|0\rangle + \frac{2\sqrt{2}+3i}{9}\,|1\rangle$.
   (c) $|u\rangle = \omega\,|0\rangle + \omega^2\,|1\rangle$ and $|v\rangle = |-\rangle$ where $\omega = \sqrt[8]{1}$ is the eighth root of unity.
3. State whether the following state pairs form a basis for the Hilbert space $\mathcal{H}_2$?
   (a) $|\psi_1\rangle = \sqrt{\frac{3}{2}}\,|0\rangle + \frac{1}{2}\,|1\rangle$, $|\psi_2\rangle = -\frac{1}{2}\,|0\rangle + \sqrt{\frac{3}{2}}\,|1\rangle$.
   (b) In vector notation, $x_1 = [1, 0]^T$ and $x_2 = \frac{1}{\sqrt{2}}[-1, 1]^T$.
   (c) $|\psi_1\rangle = \frac{\sqrt{3}+i}{2}\,|0\rangle - \frac{1+i\sqrt{3}}{2}\,|1\rangle$, $|\psi_2\rangle = \frac{\sqrt{3}+i}{2}\,|0\rangle + \frac{1+i\sqrt{3}}{2}\,|1\rangle$.
4. Find the norm of the following:
   (a) $|u\rangle = \left(1 - \sqrt{3/4}\right)|0\rangle + \sqrt{i/4}\,|1\rangle$
   (b) $|u\rangle = \omega\,|0\rangle + \omega^2\,|1\rangle$
   (c) $|u\rangle = \frac{\sqrt{3}}{2}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$
5. Express the given states in $\{|+\rangle, |-\rangle\}$ basis and the $|\psi_1\rangle, |\psi_2\rangle$ basis from question 1.
   (a) $|\psi\rangle = \frac{1}{2}\,|0\rangle + \frac{\sqrt{3}}{2}\,|1\rangle$
   (b) $|\psi\rangle = \frac{1}{\sqrt{3}}\,|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}\,|1\rangle$
   (c) $|\psi\rangle = -i\frac{\sqrt{15}}{4}\,|0\rangle + \frac{1}{4}\,|1\rangle$
6. What will be the outcomes if the following states are measured in the standard basis and the Hadamard basis repsectively?
   (a) $\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle$
   (b) $\frac{1}{\sqrt{2}}\,|+\rangle + \frac{1}{\sqrt{2}}\,|-\rangle$
   (c) $\frac{1+i}{2}\,|+\rangle + \frac{1-i}{2}\,|-\rangle$
   (d) $\sqrt{2}\,|+\rangle - |0\rangle$
7. Compute the probability of Bob measuring the state $|1\rangle$ in analysis of the BB92 protocol. Also, complete the analysis for the case where Alice has the random bit a set to be 1.