Due: 40 mins
Date: February 20th, 2025

Quiz (Repeat) (10 points/5% weight)

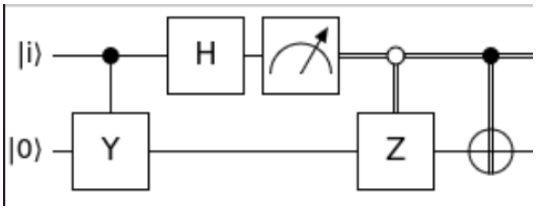
Common states: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$, $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)$.

Common gates: H maps $|0\rangle$ to $|+\rangle$ and $|1\rangle$ to $|-\rangle$, X maps $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$, CNOT maps $|a\rangle\,|b\rangle$ to $|a\rangle\,|a \oplus b\rangle$ for $a, b \in \{0, 1\}$, Z maps $|0\rangle$ to $|0\rangle$ and $|1\rangle$ to $-\,|1\rangle$.

Problem 1. *(Points :3)*

1. Write down the matrix for $R_Y(\theta)$.
2. Write down the matrix for $e^{i\theta X}$.
3. Write down the matrix for $\sqrt{R_Z(\theta)}$.

Problem 2. *(Points :2+1=3)* Consider the following circuit. Here $|i\rangle$ stands for the $|+i\rangle$ state. Observe that the control of the second controlled-gate is reversed, i.e., Z is applied when the control is 0. The third control gate applies the X gate to the target.



Answer these questions by showing all calculations.

1. Write down the output states along with their probabilities.
2. Write down the density operator of the second qubit *just* after the measurement operation.

Problem 3. *(Points :1+1+2=4)* Consider a two-qubit quantum circuit that first applies a CNOT using the first qubit as control, then applies an X on the first qubit, and finally applies a CNOT using the first qubit as control.

1. Show the output of the circuit on $|10\rangle$.
2. Show that the circuit implements the operator $X \otimes X$.
3. Instead of applying X on the first qubit, suppose X was applied to the second qubit. What operator does this new circuit correspond to? Explain.
