Worksheet 4
IQC 2024

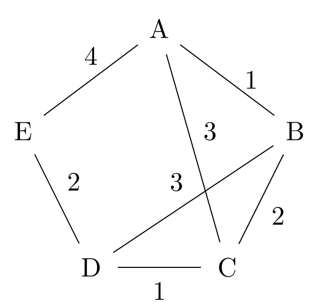
19 Mar 2024

1. Recall from the lecture that we defined the Hamiltonian for finding the Max-Cut of a graph as
$$H_1 = \frac{1}{2}\Big[(\mathbb{I} - Z_a Z_b) + (\mathbb{I} - Z_b Z_d) + (\mathbb{I} - Z_d Z_c) + (\mathbb{I} - Z_b Z_c)\Big].$$
Show that H_1 is indeed Hermitian.

2. In the lecture, we saw how we could compute the expectation $\langle\psi| Z_1 Z_2 |\psi\rangle$ of the Hamiltonian $Z_1 Z_2$ when the state is $|\psi\rangle$ if we have a two-qubit system. Now, explain how you would compute the expectation of the following three-qubit Hamiltonians when we have a three-qubit system.
   (a) $H_2 = \frac{4}{7} Z_1 Z_2 - \frac{2}{3} Z_2 Z_3$
   (b) $H_3 = \frac{2}{5} Z_3 Z_2 - \frac{1}{2} Z_2 Z_3 Z_1$

3. Construct Hamiltonians to solve the Boolean satisfiability problem for the following instances.
   (a) $(x_1 \vee x_2) \wedge (x_2 \vee x_3)$
   (b) $(x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee x_4 \vee \overline{x}_4) \wedge (x_1 \vee x_2 \vee x_3)$
   (c) $(x_1 \vee x_2 \vee x_4 \vee \overline{x}_3 \vee x_5) \wedge (\overline{x}_1 \vee \overline{x}_2) \wedge (x_5)$

4. Consider the following weighted graph G. Construct a Hamiltonian to find the Maximum weighted cut of this graph. **Note:** The weight of a cut of a weighted graph is the sum of the weights of the edges crossing the cut.



5. In the class we saw that one way to approximate the exponential $e^{-i\Delta t(A+B)}$ is as $e^{-i\Delta t(A+B)} = e^{-i\Delta t A} e^{-i\Delta t B} + E$ where $\|E\|_{op} = O(\Delta t^2)$. Yet another approximation can be given as
$$e^{-i\Delta t(A+B)} = e^{-i\frac{\Delta t}{2}A} e^{-i\Delta t B} e^{-i\frac{\Delta t}{2}A} + E'$$
where $\|E'\|_{op} = O(\Delta t^3)$.
   (a) If we were to use the latter approximation, what would be number of queries made to the U_f when solving the unordered search problem using Hamiltonian simulation?
   (b) There are other slightly complicated approximations that approximate $e^{-i\Delta t(A+B)}$ with $O(\Delta t^p)$ for any constant $p \in \mathbb{N}$ such that $p \geq 3$. What would the query complexity be in this case?