HW3 (25 points)

Problem 1. *(Points :4)* Suppose $f : \{0,1\}^n \to \{0,1\}$ is an n-bit Boolean function that is either the constant 0-function or a balanced function, and further suppose that you are given a quantum oracle U_f to make queries to f. Design an *amplitude-amplification* based zero-error (exact) algorithm to identify which one it is. The algorithm should use $O(1)$ queries.

Specify all details of the algorithm, namely what operators should be applied and in what order, by preferably drawing a circuit. Explain the idea of the algorithm and analyse the number of queries made by it. The algorithm may use ancilla.

Problem 2. *(Points :1+4=5)* Consider the following version of Grover iterator: $G = AR_0A^\dagger R_g$
where

- $R_0 = ((1 - e^{i\pi/3})\,|0^n\rangle\langle 0^n| - I)$
- $R_g = (I - (1 - e^{i\pi/3})\,|\psi_g\rangle\langle\psi_g|)$

(a) Show that the probability of observing a bad state after G is applied on $A|0^n\rangle$ is ϵ^3 where ϵ is the probability of observing a bad state if $A|0^n\rangle$ is measured.

(b) How does this algorithm compare to the Grover iterator we studied in the lecture concerning the reduction of error?

Problem 3. *(Points :1+4=5)* Consider the 2-qubit Hamiltonian $H = Z \otimes X$ and let $t = 2\pi/3$.

(a) Consider the operator $U = e^{-iHt}$. Show that U is unitary.

(b) Design a quantum circuit to implement U. One way to implement any operator is to consider its action on some basis – the trick is to choose this basis carefully. You can use ancilla if you want, but try to minimise their use; however, if you must use, you should reset their state at the end. Draw the circuit and explain its step-by-step behaviour.

Problem 4. *(Points :3)* Consider the following 2-qubit feature map/kernel $U(x_1, x_2) = CNOT \cdot (R_X(x_1) \otimes R_X(x_2))$, where x_1 and x_2 are normalized real numbers between 0 and 2π representing the features. This kernel creates feature states that, at least structurally, appear to be entangling and involving superposition. There is a general hypothesis that it would be better to create states that are entangled; thus U should serve the purpose.

However, prove that the states produced by U are not effectively entangling. To do that, construct another kernel **without** involving any 2-qubit gate that would produce the same separating hyperplane as when U would be used. *Hint: This is a question of QKE – quantum kernel estimation.*

Problem 5. *(Points :8)* In this question, you will construct a model for a classification task using quantum kernels. You can refer to this link for a construction: https://qiskit-community.github.io/qiskit-machine-learning/tutorials/03_quantum_kernel.html. Two primary primitives required to construct a quantum kernel are a feature map and a subroutine to compute fidelity.

1. Your task in this question is to construct three custom feature maps - one each with no entanglement, linear entanglement and full entanglement. You are to construct custom feature maps that are not constructed using the available feature maps classes (please see https://docs.quantum.ibm.com/api/qiskit/circuit_library#data-encoding-circuits for the readily available feature map classes).
2. For each of these feature maps, use the constructed feature map and a fidelity computing subroutine to define a quantum kernel.
3. Using each of these kernels, use an 80:20 split of data in `data.txt`, learn from the 80% data and test against the other 20% points. Compare the accuracy score obtained across each of these feature maps.
4. Finally, using each of the kernels, classify the test data given in `data-test.txt` and output their labels as output by the model.

Note: Each data point in `data.txt` has four features followed by a 0/1 label.
