CSE622 W23/Quantum Computing: Homework 1

Due date: 4th Feb 2023, 11:59pm

Announced: 24th Jan, 2023 (25 points

USE BRA-KET notation as much as possible.

1. (2 points) Show that the Y operator, with Hadamard operators applied before and after, is identical to the Y operator (upto a global phase), i.e., $HYH = -Y$. (Do not perform matrix multiplication.)

1. (2 points) Show that $XYX = -Y$ (avoid matrix multiplication) and use this to prove that $XR_y(\theta)X = R_y(-\theta)$.

2. (3 points) Express the operator $XHYZ$ as a linear combination of linear operators $\{|i\rangle\langle j|\}_{i,j=0,1}$.

3. (2 points) Describe an operator G (either as action on basis states or as a linear combination of linear operators) such that $G^2 = H$.

4. (3 points) Let $\{|1\rangle, \ldots, |d\rangle\}$ denote the standard basis for a d-dimensional Hilbert space and let U be a unirary operator. Therefore, we get another basis $\mathcal{B}$ with states like $|\alpha_i\rangle = U|i\rangle$. Show that measuring any state $|\psi\rangle$ in the $\mathcal{B}$ basis is same as first applying U to $|\psi\rangle$ and then measuring in the standard basis (same implies that the probability of different possibilities are identical, and so are the post-measurement states). This will show that if we have an apparatus for measuring only in the standard basis, then we can effectively measure in any other basis.

5. Let θ be a fixed, known angle. Suppose someone flips a fair coin and, depending on the outcome, either gives you the state $|0\rangle$ or $\cos\theta|0\rangle + \sin\theta|1\rangle$ (but does not tell you which).

(a) (3 points) Consider measuring the given state, say $|\psi\rangle$, in the orthonormal basis consisting of $|\alpha\rangle = \cos\phi|0\rangle + \sin\phi|1\rangle$ and $|\beta\rangle = \sin\phi|0\rangle - \cos\phi|1\rangle$. Find the probabilities of all the possible measurement outcomes for each possible value of the given state.

(b) (3 points) Find a strategy to predict the state of $|\psi\rangle$. Calculate the probability of your strategy being correct.

(c) (3 points) Calculate the optimal value of $|\phi\rangle$ in order to best distinguish the states. What is the optimal success probability?

6. (6 points) Create an account on IBM Quantum. Then perform the following experiments on any quantum backend, not the simulator (note that there is often a large queue for the backends, so if you submit your jobs at the last minute, your program may not finish within time).

(a) (3 points) What are the eigenstates of Pauli-Y? Verify experimentally. Attach pictures of your quantum circuit, explain the experiment process, attach output of experiments, and discuss the results. You should note that current QPUs are noisy, and hence may not always give the exact results.

(b) (3 points) Qiskit uses a parameterized $U(\theta, \phi, \lambda)$-gate as the most general single-qubit unitary gate. It is defined by:

$$U(\theta, \phi, \lambda) = \begin{bmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2} \end{bmatrix}$$

First, write Pauli-Y gate as a $U(\theta, \phi, \lambda)$ gate, and then repeat the first experiment using this gate instead of directly using the Pauli-Y gate. Did the errors increase or decrease?