

CSE525 Lec2: Recursion

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Stoogesort

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STOOGESORT(A[0..n-1]):  
1  if n = 2 and A[0] > A[1]  
2      swap A[0] ↔ A[1]  
3  else if n > 2  
4      m = ⌈2n/3⌉  
5      STOOGESORT(A[0..m-1])  
6      STOOGESORT(A[n-m..n-1])  
7      STOOGESORT(A[0..m-1])
```

After 4.	U ₀	V ₀	W ₀
After 5.	U ₁	V ₁	W ₁ =W ₀
After 6.	U ₂ =U ₁	V ₂	W ₂
After 7.	U ₃	V ₃	W ₃ =W ₂

3 partitions of A: U V W

Proof by induction on the length of A.

Induction hypothesis: ???

Prove that: Assuming IH, U₃, V₃ & W₃ are individually sorted.

Prove that: Assuming IH, all x in U₃ ≤ all y in V₃ ≤ all z in W₃.

Thm: StoogeSort is correct.

Q: What is its worst-case complexity ?

7, 8, 9 4, 5, 6 1, 2, 3

Prove that: Assuming IH, U3, V3 & W3 are individually sorted.

After line 5, U1 is in increasing order, and so is V1.

After line 6, V2 is in increasing order, and so is W2.

U2 remains in increasing order.

After line 7, U3 is increasing order, and so is V3.

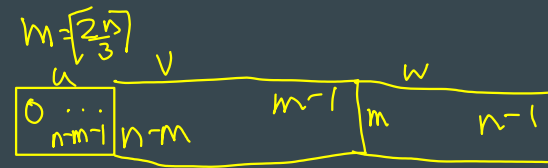
W3 = W2 remains in increasing order.

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```

After 4.	U0	V0	W0
After 5.	U1	V1	W1=W0
After 6.	U2=U1	V2	W2
After 7.	U3	V3	W3=W2

Prove that: Assuming IH, all x in $U_3 <$ all y in $V_3 <$ all z in W_3 .



After line 7, since U_3 and V_3 are sorted, It means all e in $U_3 <$ all e' in V_3 .

If we could prove all e' in $V_3 <$ all e'' in $W_3=W_2$, we are done.

Each e' in V_3 either belongs from $U_1=U_2$ or from V_2 . There can be two cases.

1. If e' belongs to V_2 , since all elements in $V_2 <$ all elements in W_2 , so, $e' <$ all elements in W_2 .

2. If e' belongs to $U_2=U_1$, then we have few further cases.

Let $\min V_1 =$ minimum element in V_1 . Since e' is in U_1 , $e' <$ $\min V_1$.

- If $\min V_1 \leq$ minimum in W_2 , all elements in $W_2 >$ $\min(V_2) \Rightarrow \min V_1 >$ e'
- If $\min V_1 >$ $\min(W_2)$, then all of $V_1 \geq \min V_1 >$ $\min(W_2)$, so W_2 should contain all elements of V_1 and $\min(W_2)$. But this is a contradiction since $|W_2| \leq |V_1|$.

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```

Intuition
 $n=10$ $m=7$ StoogeSort([0 ... 6]), StoogeSort([3 ... 9]), StoogeSort([0 ... 6])
 $U, V, W:$ [0 ... 2] [3 ... 6] [7 ... 9]

Generally, $U, V, W : [0 \dots n-m-1] [n-m \dots m-1] [m \dots n-1]$
 $|U| = n-m \leq n/3, \quad |W| = n-m \leq n/3, \quad |V| = n - |U| - |W| \geq n/3$

proof ↙

After 4.	U_0	V_0	W_0
After 5.	U_1	V_1	$W_1=W_0$
After 6.	$U_2=U_1$	V_2	W_2
After 7.	U_3	V_3	$W_3=W_2$