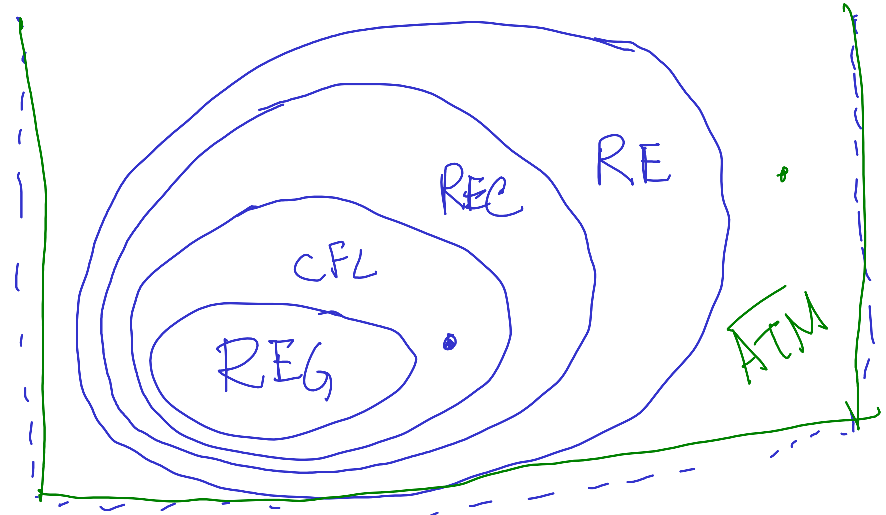


CSE322 Theory of Computation

Lec 21

REC (Decidable) is a proper subset of RE (Recognizable).

Polynomial Solvability is recognizable but undecidable.

Natural numbers N=0,1,2,3,... are countable.

Any set that has a 1-1 correspondence with N is countable.

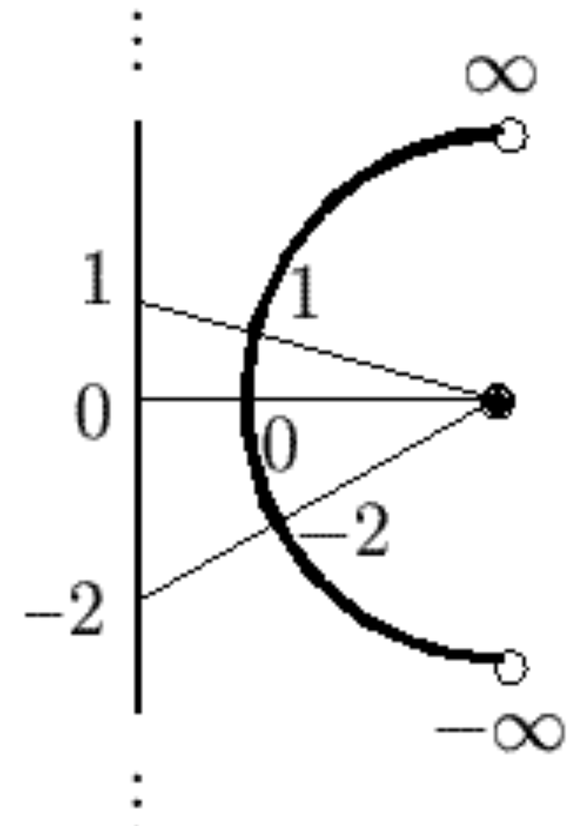
S is countable if one can define "get y-th element of S" for any natural num. y and for every x in S, x is the y-th element for some y.

Are even numbers countable?

Are the final scores in TOC of all students countable?

Are the rational numbers countable?

Are the set of infinite strings countable?



ALLDFA = {w : w is an encoding of a DFA over alph.}

ALLTM = { w | w is an encoding of a TM over alph.}

Claim: ALLDFA and ALLTM are countable sets.

Q: Define "get y-th TM over alph." How?

```
def getTM(k): // get k-th TM
    counter = 0
    for every string w in string order:
        if w does not encode a TM: ignore
        else, counter++
        if counter==k: output w.
```

ALL : class of every language over any alph.

000

$w_4 \notin L$ if $w_4 \in L_4$

Claim: ALL is uncountable. ALL = { L | L is a language over some alph.}

$$
\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & & & \\
\Sigma^* = \{ & \varepsilon, & 0, & 1, & 00, & 01, & 10, & 11, & 000, & 001, & \cdots & \} ; \\
A = \{ & & 0, & & 00, & 01, & & & 000, & 001, & \cdots & \} ; \\
\chi_A = & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \cdots & .
\end{array}
$$

characteristic function

$w_4 \in L$ if $w_4 \notin L_4$

	w_1	w_2	w_3	w_4	w_5	...
L_1						
L_2						
L_3						
L_4						
⋮						

Assuming an ordering that defines the 'i'-th language in ALL, design a language L which does not belong to ALL.

$$\chi_L(w_i) = \begin{cases} 0 \text{ if } w_i \in L_i \Rightarrow \chi_{L_i}(w_i) = 1 & w_i \notin L \\ 1 \text{ if } w_i \notin L_i \Rightarrow \chi_{L_i}(w_i) = 0 & w_i \in L \end{cases}$$

ALL $\{ L_1, L_2, L_3, L_4, \ldots \}$

For any alphabet, number of languages more than number of possible TMs.

ALLTM is strictly smaller than ALL.

{ <M> : M is a TM } vs { L : L is a language of some TM }

ALLTM = { L | L is a language of some TM}

Halting Problem

Suppose HALT = { ⟨M,w⟩ : M halts on input w } is decidable (by TM D).
Diagonalize against all TMs on self-inputs.

This is the second method for proving undecid.

Run D(⟨U, ⟨U⟩⟩)

Behaviour of U(⟨U⟩) is contradictory!

w=

Construct TM U:

U(w):

1. if w is not TM encoding, rejects (accepts ?)
2. if w is TM encoding:
3. w = ⟨M⟩ = ⟨U⟩
4. Run D(⟨M,w⟩) D(⟨M, ⟨M⟩⟩)
5. If D accepts ⟨M,w⟩, // M halts on w
6. U(w) loops
7. If D rejects ⟨M,w⟩, // M doesn't halt on w
8. U(w) accept

Decidable and Recognizable lang.

Thm L is decidable iff L is recognizable and L-complement is recognizable.

Proof:-

=> Suppose L is decidable. Then $L \in RE$.

$\Rightarrow \bar{L} \in REC$. $\therefore \bar{L} \in RE$.

<= Suppose M recognizes L and M' recognizes L'.

Construct decider D for L.

D(x):

Runs M and M' parallely on x.

If M accepts, D accepts

If M' accepts, D rejects.

Post's Correspondence Problem

bba	ab	bba	a
bb	aa	bb	baa

1 2 3 4

2 3 4 2 1

ab bba a ab bba

aa bb baa aa bb

Not in syllabus: Given <M,w>, construct a set of cards such that...

- if M accepts w then the cards have a matching
- if M does not accept w then the cards do not have a matching

Unrecognizable language: A-TM-complement

Proof: Proof by contradiction.

Suppose A-TM' is recognizable.

$$\overline{A\text{-}TM} = \{ \langle M, w \rangle \mid M \text{ doesn't accept } w \}$$

We know that A-TM is recognizable.

$\therefore$ A-TM would be decidable.

This is a contradiction.

Ex:- $RE \cap CoRE = REC$

← trivial from def.

→ earlier thm.

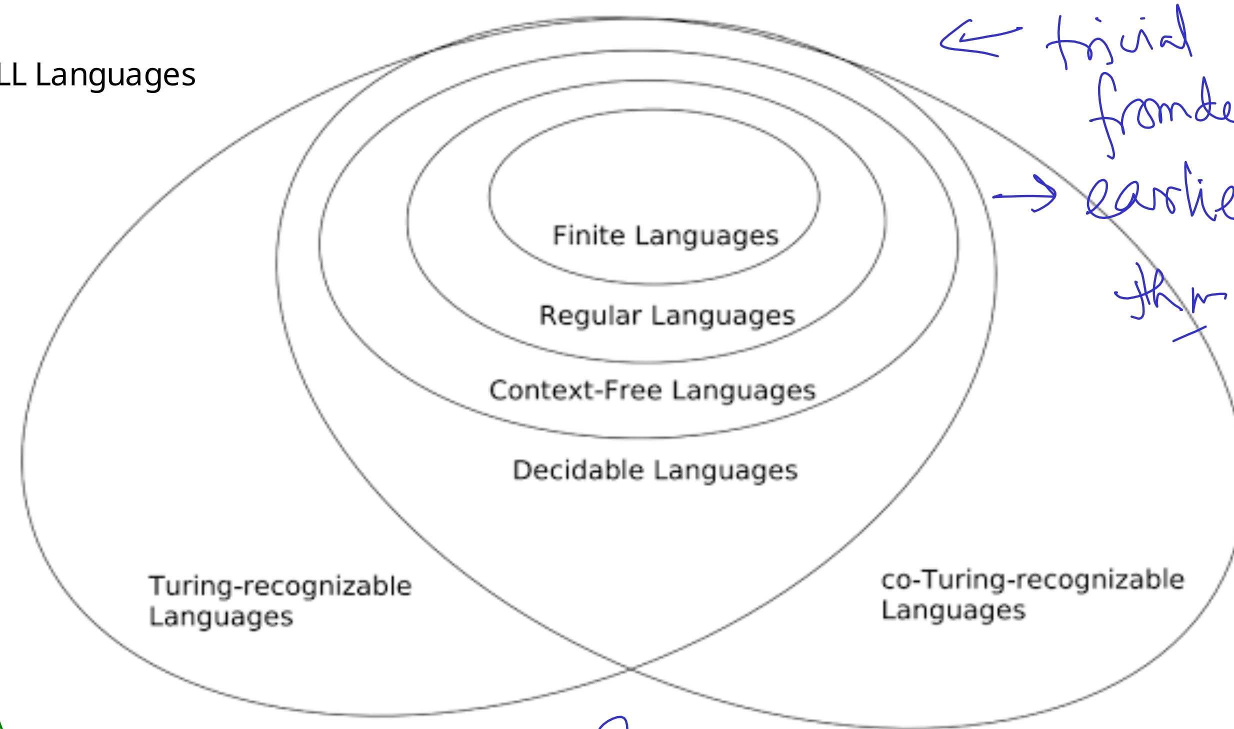
Complement Classes

co-C =

{ L : complement(L) in C }

$L \in CoC \leftrightarrow \overline{L} \in C$

$\overline{L} \in CoC \leftrightarrow L \in C$



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co-R.E. = co-Recognizable

= { L : complement of L is R.E./recognizable }

$\overline{(\overline{ATM})} = ATM \in RE$

$\overline{ATM} \in CoRE$

Thm: $DEC \subseteq CoRE$

If $L \in DEC$ then $L \in CoRE$

Proof:- $L \in DEC \rightarrow \overline{L} \in DEC$

$L \in CoRE \leftarrow \overline{L} \in RE$

$DEC = CoDEC = \{ L : \overline{L} \in DEC \}$

$L \in DEC \leftrightarrow \overline{L} \in CoDEC$

$\updownarrow$

$\overline{L} \in DEC$