CSE322 Theory of Computation (L17)

$\langle \{q_0, q_1, q_2\}, \{0,1\}, \{(q_0,0,q_1), (q_0,1,q_0), \ldots\}, q_2, \{q_1, q_2\} \rangle \# 0110$

Decidable and Recognizable

In tutorial, you have created a TM to simulate a DFA D on an input w, given both the string encoding of D and w.

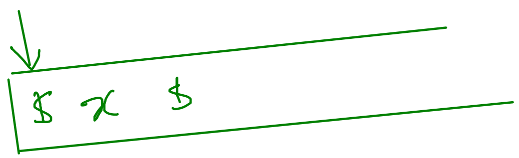
TMs as subroutines

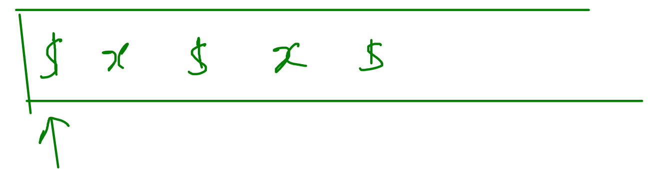
string

tries to never moves head left of the first \$

CopyTM   qcopy0 \$ x \$ $\vdash^*$ qcopydone \$ x \$ x \$

Initial ID   Final ID





y qcopy0 \$ 011 \$ $\vdash^*$ y qcopydone \$ 011 \$ 011 \$

EqualTM   qeq0 \$ x \$ y \$ $\vdash^*$ qeqsame \$ x \$ y \$ if x = y

$\vdash$ qeqdiff \$ x \$ y \$ if x != y

M

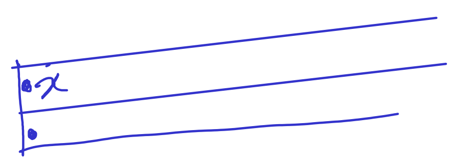
δ

qeq0, qeqsame,

$\delta_N(qM0, t_1, t_2) = (qM1, t_1, a, S, D)$

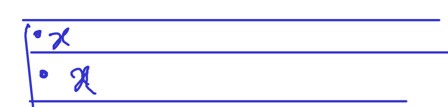
$\delta_M(qM0, t_2) = (qM1, a, D)$

Let M=<Q,..., qM0, qMa, qMr> be some 1-tape TM.



N(x): // two tape TM

1. Copy x on the second tape and put second head on the leftmost cell.



2. Run M on the second tape.
3. If M goes to qMa, N goes to qrej. If M goes to qMr, N starts looping.

$\delta_N(qMa, t \in \Gamma) = (q^N_{rej}, \ldots)$

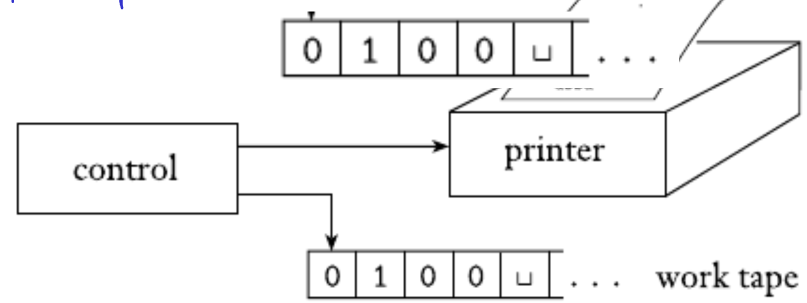
$\delta_N(qMr, t \in \Gamma) = (qMr, \ldots)$

Recursively Enumerable

2 tape, two heads

work tape

output tape, head is rightonly.



Enumerator

L is recursively enumerable if there is a two-tape TM that (ignores input) ...

① * If w (delimited by #) is written any time on tape-2, then w is in L

* For all w in L, w appears at some point on tape -2 (possibly with repetitions),

② separated by #

011# 1011# 01# 011 # . . .

$L = L(TM)$

Thm: L is recognizable iff L is recursively enumerable

Proof:

$L = L(E)$

Proof of Claim ①

Suppose $x \in L$.

∴ E prints x at some point, whenever E prints x, tape in Line 2 will succeed & M will accept.

<= Suppose enumerator E outputs L. Build M to recognize L.

3 tape M/c

$M(x)$: // If $x \in L$ then M should accept ①

If $x \notin L$ then M shouldn't accept ≡ If M accepts x then $x \in L$. ②

1. Runs E on the 2nd & 3rd tape

2. Whenever E outputs some strings $w\#$ on 3rd tape,

if $w = x$ goto q_{acc}, else continues with E.

Proof of Claim ②

Suppose M accepts x,

There must be x on E's output tape. $x \in L(E) = L$.

Thm: L is recognizable iff L is recursively enumerable

Proof:

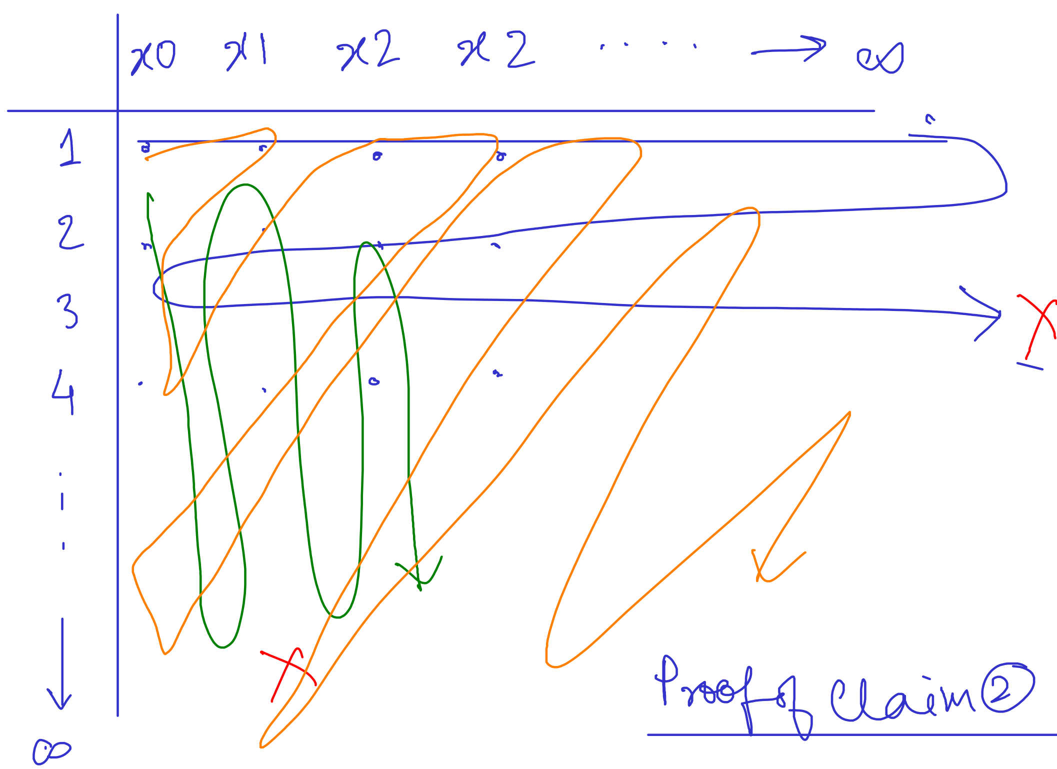
=> Suppose M recognizes L. Build enumerator E to output L.

Use TMs with a counter and dovetailing.

E( ): ① If $x \in L$, E should print x.
② If E prints x, then $x \in L$.

1. For every string x in dictionary order
2. Run M on x on 1st tape.
3. When M accepts x, E prints x,#

$\notin L$	$\notin L$	$\in L$	$\notin L$	$\in L$	$\in L$	
x_0	x_1	x_2	x_3	x_4	x_5	...



Proof of claim ①

Suppose $x \in L$. ∴ M accepts x within some steps, denoted n. When E has (n, x) on its counter tape, M will accept x. E never lets M go into an infinite loop.

Proof of claim ②

Suppose E prints x.
∴ M must have accepted x.
∴ $x \in L$.