CSE322 Theory of Computation (L8)

Today

Review

$w : \exists w' \in L$ s.t. switch$(w') = w$

Define Switch3(L) = { w such that L contains w' &
w is a version of w' in which
every alternate 1 is switched to zero,
starting from the first 1}.

Show that regular languages are closed under Switch3.

N

D accepts L

$\langle Q, \Sigma, \delta, q_0, F \rangle$

switch (w) = every 1 at odd index / pos in switched to 0

N accepts w iff D accepts w' s.t. switch(w') = w.

⇒ N simulates D on a guess of w!

L = { 0, 1, 11, 011, 10101, ... }

switch(L) = { 0, 0, 01, 001, ... }

w = 010010 010

0 1

0 1

w' = 010

1

011... ⇒ 001...

111 ⇒ 010...

1st 1

011 ⇒ 000

111 ⇒ 010

any 0 in w ⇒ 0 or { 1 in w' only when that 1 is in some odd position in w'

stored in a state.

States = Q x {ODD,EVEN} // EVEN/ODD keeps track of the position of 1 in w'
* State is (q,ODD) means that the last 1 was an odd 1 in w',
  so it must have been switched to 0, and the current state in D on w' is q
* State is (q,EVEN) means that the last 1 was an even 1 in w',
  so it must have been retained as 1, and the current state in D on w' is q

(q, ODD) (q, EVEN)

the position of the last 1 of the guessed w'

Starting state = (q0, EVEN) //no ones has been seen until now

Final states = F x {ODD,EVEN}

$\delta 1((q, odd), 1) =$

Transition function:
* d1 ((q,ODD),1) = { (d(q,1),EVEN) } // last 1 was an odd one in w',
  so it must have been switched to 0; now another 1 is observed,
  thus move according to D

DFA

$q0 \xrightarrow{w'} q \xrightarrow{?1} (\delta(q,1), Even)$

w: 1

* d1((q,EVEN),1) = {} // last 1 was even one in w' so it must have been retained as 1 in w;
  now, another 1 is observed in w – however, it is not possible have another 1 in w' retained as 1,
  so go to the NULL-state.
* d1((q,ODD),0) = { (d(q,0),ODD) } // last 1 was even one in w' so it must have been retained as 1 in w;
  now a 0 is observed in w' which could not have come from a 1 in w
  (since that 1 would be an even 1 which would not be flipped) –
  hence, that 0 in w must have come from a 0 in w'
* d1((q,EVEN),0) = { (d(q,0),ODD), (d(q,1),EVEN) } // last 1 was an odd one in w',
  so it must have been switched to 0; now a 0 is observed, guess both options:
  0 in w' is retained as 0 in w, and 1 in w' (it would be even one) is switched to 0 in w

Pumping Lemma game

You (prove language to be non-regular).

Adversary

give a p →

← p

choose $w \in L, |w| \geq p$

w →

← x, y, z

show $xy^kz \notin L$

k →

Agree that PL is violated

Prove non-regularity of ... $0n1n = \{0^n 1^n : n \geq 0\}$

$PAL = \{ w : w = rev(w) \}$ $w \in \{0,1\}^*$

$BAL = \{$ balanced strings using (and) $\}$

pumping length p

$1\,0\,\boxed{0 \cdots}\,0\,1$ (y)

$w = 1 0^{p-2} 1 \in PAL$

Case: $y = 10^{p-2}1 \rightarrow$

Case: $y = 10^t$

Case: $y = 0^t 1$

Case: $y = 0^t$, $x = 10^a$, $z = 0^b 1$, $xy^0z = 10^a 0^b 1 \in PAL$

$\Sigma = \{0,1\}$

$WW = \{ ww : w \text{ is a string} \}$

$LL = \{ tt : t \text{ is a string} \}$

(a) $L = \{a^n b a^{2n} : n \geq 0\}$

(b) $L = \{a^i b^j : i = j \text{ or } 2i = j\}$

$w = 1^p 0 1^p 0 \in LL$

$x = 1^a$, $y = 1^b$, $z = 1^c 0 1^p 0$

$xz = 1^{a+c} 0 1^p 0 \notin LL$

$a + c < p$

$\nexists w' w'$

$\Rightarrow w'$ must end with 0

$w' = 1^{a+c} 0$ or $1^p 0$

$w = 1^p 0 1^p$

$|xy| \leq p \Rightarrow xy = 1^k \Rightarrow y = 1^k, k \geq 1$

$|y| \neq 0$

$1\,1 \cdot \boxed{y \cdot}\,1\,0\,1\,1 \cdots 1$ (p, $1k$, p)

Otherwise $w' = 1^{p-k} 0 1^p \notin PAL$

$(p-k+1)$ Symbol in $w' = 0$

$(p-k+1)$ th " $rev(w) = 1$

(c) $L = \{ x : x$ begins with a non-null string of the form $ww \}$

$w = a^p b a^{2p}$ $y = a^t$ $xz = a^{p-t} b a^{2p}$

$L = \{a^n b a^{2n} : n \geq 0\}$

$L = \{a^i b^j :$ either i is a multiple of j or j is a multiple of $i\}$

$w = a^p b^p \rightarrow$ not possible if $t > p$ $|xy| \leq p \Rightarrow y = a^t$ $1 \leq t \leq p$,

Try: $w = a^{2p} b^{2p}$

$BAL = \{ w : w \text{ is a balanced string} \}$ $\quad ((((\,))) (\,) ((\,)) (\,))((\,))$

$\Sigma = \{ (,) \}$

$w = \underbrace{((\cdots(}_{p}\underbrace{)))}_{p}\cdots)$

$w \in L$

$\cancel{w = (^p\,)\, (^p} \notin BAL$

$\cancel{pal(w)} \in BAL$

$w = \cancel{(^{p/2}}\,)^{p/2}$

$\therefore y = (^k$

$x y^0 z = (^{p-k}\,)^p \notin BAL$

$\#((, w) = p-k$

$\#(), w) = p$