

CSE322 Theory of Computation (L8)

Today

Review

Define $\text{Switch3}(L) = \{ w \text{ such that } L \text{ contains } w' \text{ \& } w \text{ is a version of } w' \text{ in which every alternate 1 is switched to zero, starting from the first 1} \}.$

\downarrow
N

Show that regular languages are closed under Switch3.

$w : \exists w' \in L \text{ st. } \text{switch}(w') = w$

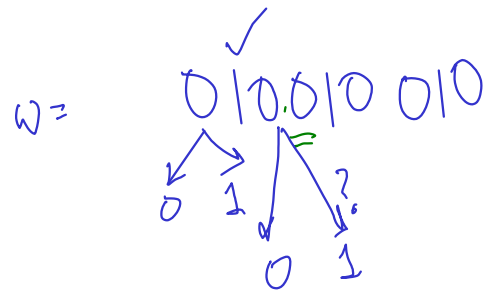
D accepts L
 $\hookrightarrow = \langle Q, \Sigma, \delta, q_0, F \rangle$

$\text{switch}(w) = \text{every 1 at odd index / in switched to 0}$

N accepts w iff D accepts w' st. $\text{switch}(w') = w.$

$L = \{ 0, 1, 11, 011, 10101, \dots \}$
 $\text{Switch}(L) = \{ 0, 0, 01, 001, \dots \}$

\Rightarrow N simulates D on a guess of w'.



w' = 010
 1

011... \Rightarrow 001...
 111 \Rightarrow 010...

$\xrightarrow{1st}$
 011 \Rightarrow 000
 111 \Rightarrow 010

any 0 in w \Rightarrow 0 or $\left\{ \begin{array}{l} 1 \text{ in } w' \\ \text{only when that 1 is in some odd} \\ \text{position in } w' \end{array} \right.$

← stored in a state.

States = $Q \times \{ODD, EVEN\}$ // EVEN/ODD keeps track of the position of 1 in w'

- * State is (q, ODD) means that the last 1 was an odd 1 in w' , so it must have been switched to 0, and the current state in D on w' is q
- * State is $(q, EVEN)$ means that the last 1 was an even 1 in w' , so it must have been retained as 1, and the current state in D on w' is q

(q, ODD) $(q, EVEN)$
 \uparrow
 the position of the last 1 of the guessed w'

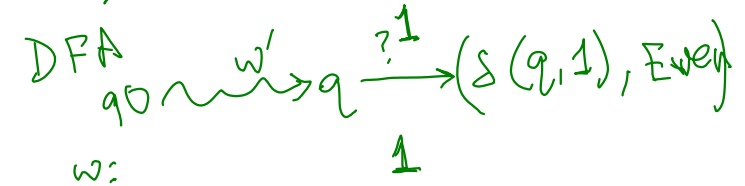
Starting state = $(q_0, EVEN)$ // no ones has been seen until now

Final states = $F \times \{ODD, EVEN\}$

$\delta_1((q, odd), 1) =$

Transition function:

- * $\delta_1((q, ODD), 1) = \{ (d(q, 1), EVEN) \}$ // last 1 was an odd one in w' , so it must have been switched to 0; now another 1 is observed, thus move according to D
- * $\delta_1((q, EVEN), 1) = \{ \}$ // last 1 was even one in w' so it must have been retained as 1 in w ; now, another 1 is observed in w - however, it is not possible have another 1 in w' retained as 1, so go to the NULL-state.
- * $\delta_1((q, ODD), 0) = \{ (d(q, 0), ODD) \}$ // last 1 was even one in w' so it must have been retained as 1 in w ; now a 0 is observed in w' which could not have come from a 1 in w (since that 1 would be an even 1 which would not be flipped) - hence, that 0 in w must have come from a 0 in w'
- * $\delta_1((q, EVEN), 0) = \{ (d(q, 0), ODD), (d(q, 1), EVEN) \}$ // last 1 was an odd one in w' , so it must have been switched to 0; now a 0 is observed, guess both options: 0 in w' is retained as 0 in w , and 1 in w' (it would be even one) is switched to 0 in w

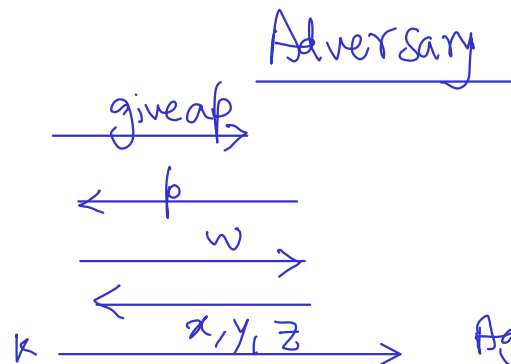


Pumping Lemma game

You (prove language to be non-regular).

choose $w \in L, |w| \geq p$

show $xy^kz \notin L$



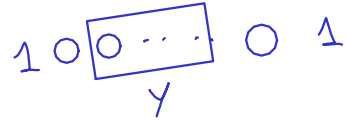
Agree that PL is violated

Prove non-regularity of ... $\{0^n 1^n : n \geq 0\}$

$PAL = \{w : w = \text{rev}(w)\}$
 $w \in \{0,1\}^*$

$BAL = \{\text{balanced strings using } (\text{ and }) \}$

pumping length p



$w = 10^{p-2}1 \in PAL$

Case: $y = 10^{p-2}1 \rightarrow$

Case: $y = 10^t$

Case: $y = 0^t 1$, $x = 10^a$, $z = 0^b 1$, $xy^0z = 10^a 0^b 1 \in PAL$

(a) $L = \{a^n b a^{2n} : n \geq 0\}$

(b) $L = \{a^i b^j : i=j \text{ or } 2i=j\}$

$\Sigma = \{0,1\}$
 $WW = \{ww : w \text{ is a string}\}$

$LL = \{tt : t \text{ is a string}\}$

$w = 1^p 0 1^p 0 \in LL$

$x = 1^a, y = 1^b, z = 1^c 0 1^d 0$

$xz = 1^{a+c} 0 1^d 0 \notin LL$
 $a+c < p$

$\Rightarrow w' \neq w'w'$
 $w' = 1^{a+c} 0 \text{ or } 1^p 0$
 w' must end with 0

$w = 1^p 0 1^p$
 $|xy| \leq p \Rightarrow xy = 1^k \Rightarrow y = 1^k, k \geq 1$
 $|y| \neq 0$

$w' = 1^{p-k} 0 1^p \notin PAL$

$(p-k+1)$ Symbol in $w' = 0$
 $(p-k+1)$ th " " $\text{rev}(w) = 1$

(c) $L = \{x : x \text{ begins with a non-null string of the form } ww\}$

$L = \{a^n b a^{2n} : n \geq 0\}$

$w = a^p b a^{2p} \quad y = a^t \quad xz = a^{p+t} b a^{2p}$

$L = \{a^i b^j : \text{either } i \text{ is a multiple of } j \text{ or } j \text{ is a multiple of } i\}$

$w = a^p b^p \rightarrow \text{not possible if } t=p$
 $|xy| \leq p \Rightarrow y = a^t \quad 1 \leq t \leq p$

Try: $w = a^{2p} b^{2p}$

$BAL = \{ w : w \text{ is a balanced string} \}$ $((((())) () (() ())) (())$

$\Sigma = \{ (,) \}$

$w = ((\dots ())) \dots)$
 $w \in L$ $\underbrace{\hspace{2cm}}_p$ $\underbrace{\hspace{2cm}}_p$

~~$w = ({}^p) ({}^p) \notin BAL$~~
 ~~$pal(w) \in BAL$~~

~~$w = ({}^{p/2})^{p/2}$~~

$\therefore y = ({}^k$

$x y^0 z = ({}^{p-k})^p \notin BAL$

$\#((, w) = p - k$

$\#(), w) = p$