

## SMIM: Generalization of High-Utility Itemset Mining (HUIM)

Table 1: Example of a transaction database and utilities of two itemsets  $\{A, C\}$ ,  $\{G, H\}$  for two different utility functions  $u()$  (as used in HUIM) and  $ucov()$  (which is subadditive and monotone).  $ucov()$  also takes a relationship graph as input as shown in Fig. 1.

TID	Transaction	$w(A)$	$w(C)$	$w(G)$	$w(H)$	$u(AC)$	$u(GH)$	$ucov(AC)$	$ucov(GH)$
$T_1$	$(A : 5) (C : 10) (D : 2)$	5	10	0	0	15	0	35	0
$T_2$	$(A : 10) (C : 6) (E : 6) (G : 5)$	10	6	5	0	16	0	40	0
$T_3$	$(A : 10) (B : 4) (D : 12) (E : 6) (F : 5)$	10	0	0	0	0	0	0	0
$T_4$	$(A : 5) (B : 2) (C : 3) (D : 2) (G : 1) (H : 41)$	5	3	1	41	8	42	20	83
$T_5$	$(B : 8) (C : 13) (D : 6) (E : 3)$	0	13	0	0	0	0	0	0
$T_6$	$(F : 1) (G : 2)$	0	0	2	0	0	0	0	0
$T_7$	$(F : 4) (G : 3)$	0	0	3	0	0	0	0	0

$u(AC) = 39$  and  $u(GH) = 42$ , so  $u(GH) > u(AC)$ ; however,  $ucov(AC) = 95$  and  $ucov(GH) = 83$ , so  $ucov(GH) < ucov(AC)$ . High-utility itemsets may change when the underlying utility function is changed.

### High-utility itemset mining (HUIM)

Given a threshold  $\theta$ , identify itemsets  $X$  with utility  $u(X) \geq \theta$ , where the utility of an itemset is defined as

$$u(X) = \sum_{T \in \mathcal{D}, X \subseteq T} u(X, T),$$

and the utility of  $X$  in a transaction is defined as

$$u(X, T) = \sum_{y \in X} w(y, T),$$

where  $w(y, T)$  denotes the weight/individual-utility of the item  $y$  in  $T$ .

### SM utility functions

- A function  $f : \mathcal{U} \rightarrow \mathbb{R}^+$  is defined as subadditive if  $\forall X, Y \subseteq \mathcal{U}, f(X \cup Y) \leq f(X) + f(Y)$ ;  $SUM(U) = \sum_{y \in U} f(y)$  is subadditive, but  $PROD(U) = \prod_{y \in U} f(y)$  is not.
- A function  $f : \mathcal{U} \rightarrow \mathbb{R}^+$  is defined as monotone if  $\forall X \subseteq Y \subseteq \mathcal{U}, f(X) \leq f(Y)$ ;  $SUM(U) = \sum_{y \in U} f(y)$  is monotone, but  $MIN(U) = \min_{y \in U} f(y)$  is not.

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### SMIM: HUIM using a subadditive & monotone utility function

Given any arbitrary subadditive and monotone (SM) utility function  $u'()$  over weighted itemsets, and threshold  $\theta$ , identify itemsets  $X$  with utility  $u'(X) \geq \theta$ , where the utility of an itemset is defined as

$$u'(X) = \sum_{T \in \mathcal{D}, X \subseteq T} u'(X, T),$$

and the utility of  $X$  in a transaction, denoted  $u'(X, T)$ , is defined in terms of the items in  $X$  along with their weights/individual utilities.

### Examples of SM functions

Let  $w(y)$  denote the weight of an item  $y$  in a set  $X$ .

- $SUM(X) = \sum_{y \in X} w(y)$  – this is used in HUIM
- Discounted profit  $DP(X)$  = total profit  $X$  under the scheme “Buy 1 pencil, get 1 eraser free”
- $Co(X)$  = number of nodes that are either in  $X$  or neighbor of some node in  $X$
- (order  $X$  in increasing order of weights)  
 $ucov(X, T) = w(y_1) \times Co(X)$   
 $+ \sum_{j=2}^k (w(y_j) - w(y_{j-1})) \times Co(\{y_j \dots y_k\})$

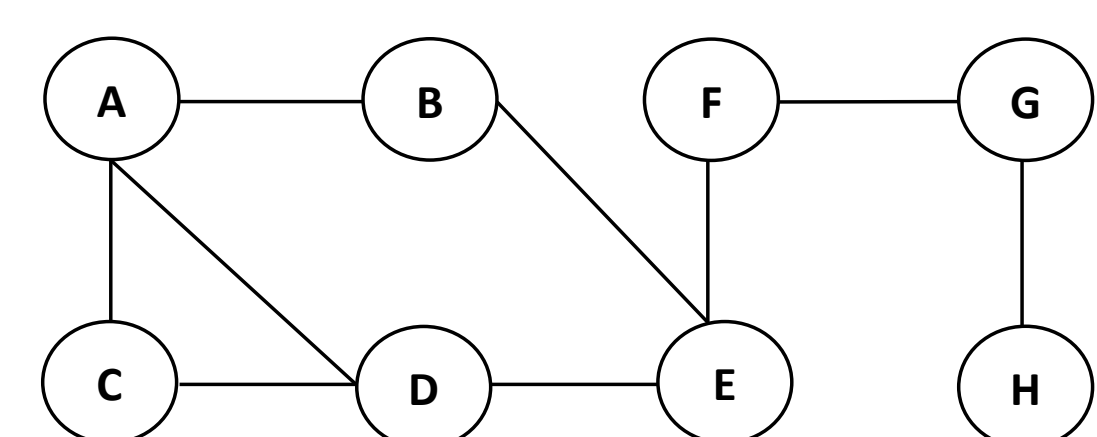


Figure 1: External graph used to compute  $Co()$  and  $ucov()$

### Contributions

- SMIM framework for mining high-utility itemsets using any subadditive and monotone utility function defined on weighted itemsets.
- SM utility functions have mathematically helpful properties, and can model traditional HUIM, HUIM in the presence of multi-item discounts, and several variations of HUIM as considered by [Yao,Hamilton,Geng 2006].
- Going beyond the HUIM framework, SMIM can incorporate extraneous interactions among the items in an itemset in their utility, e.g., to identify influential users in a Twitter dataset.
- We prove a few interesting utility functions to be subadditive and monotone, e.g.,  $DP()$ ,  $Co()$ , and  $ucov()$ .
- A novel inverted-list data structure called SMI-List and an algorithm called SM-Miner to mine high-utility itemsets for SM functions.
- We also show how to adapt the existing HUIM algorithms for SMIM, but empirically show that SM-Miner delivers better performance.

### Adapting HUIM algorithms

- Transaction merging should be disabled when adapting projection-based algorithms to SMIM.
- Ex.:  $ucov(\{F, G\}, T_6) = 7$  and  $ucov(\{F, G\}, T_7) = 15$ . If we merge them to a single transaction  $M = \{(F : 5), (G : 5)\}$ , then we get  $ucov(\{F, G\}, M) = 20$ .
- Tree-based algorithms for HUIM may be adapted towards SMIM if unpromising items are retained during local tree creation since removing them may yield incorrect estimates of some utilities.
- Ex.: Let  $T = \{(A_1 : q_1), \dots, (A_n : q_n)\}$  be some transaction, itemset  $X = \{A_1\}$  and itemset  $Y = \{A_2, \dots, A_n\}$ . Suppose that  $A_1$  is unpromising. For HUIM,  $u(X, T) + u(Y, T) = u(X \cup Y, T)$ ; therefore,  $u(X \cup Y, T) - u(X, T)$  correctly estimates  $u(Y, T)$ . However, this may not hold for other utility functions where  $f(X, T) + f(Y, T) > f(X \cup Y, T)$ .

### SM-Miner

A single efficient list-based algorithm for mining high SM-utility itemsets given blackbox access to any SM-utility function.

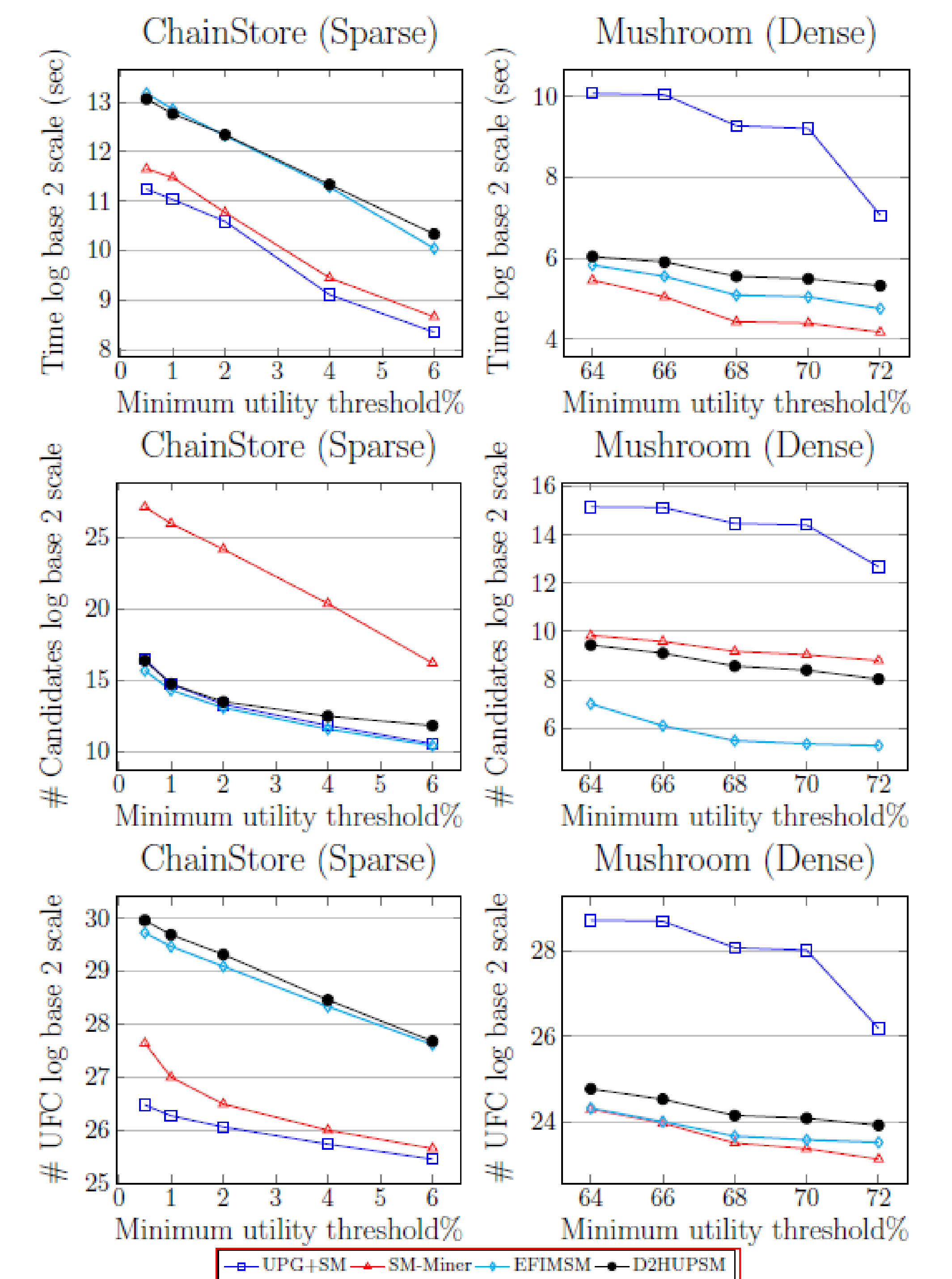


Figure 2: Performance evaluation of SM-Miner (our), and SMIM implementations of EFIM, D2HUP, UP-Growth+ (using  $ucov$ ).

- For HUIM (using  $SUM$  utility function), D2HUP and EFIMSM were observed to perform the best on sparse and dense datasets, respectively.
- Tree-based algorithm UPG+-SM and list-based algorithm SM-Miner performs better than projection-based algorithms on sparse datasets. SM-Miner competes with EFIMSM on dense datasets.
- The total execution time of the algorithms appear to be more correlated with the number of utility function calls than the number of candidates generated, unlike for HUIM.